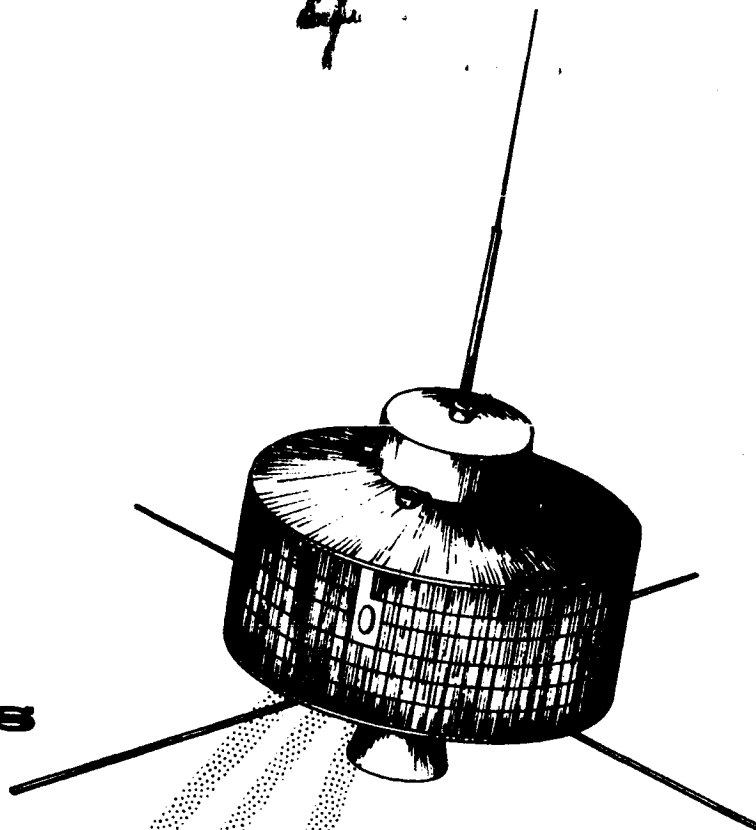


# Study Of Modulation Techniques For Multiple Access Satellite Communications



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Final Report on NASA Contract NAS 5-3544

STUDY OF MODULATION TECHNIQUES  
FOR  
MULTIPLE ACCESS SATELLITE COMMUNICATIONS

*25 August 1964*

Prepared For  
Goddard Space Flight Center  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
Greenbelt, Maryland

Federal Systems Division  
INTERNATIONAL BUSINESS MACHINES CORPORATION  
Rockville, Maryland

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## GLOSSARY

SSB-FDM	- Single Sideband-frequency division multiplexing
PN	- Pseudo-noise
T.T./N	- Test Tone-to-noise
G	- Channels/megacycle
$P_S/N_K$	- Intrinsic signal-to-noise ratio
$P_S/N_o$	- The bandwidth for which the S/N in the receiver is one
RANSAC	- Random Access Noise Signal Address Communications
$W_o$	- Voice channel bandwidth
2W	- RF bandwidth (cps; sometimes megacycles)
d	- Duty factor
K	- Number of channels
T	- Signal duration or integration time per decision
M	- Order of the alphabet ( $2^{m_b}$ )
$m_b$	- Number of bits in voice sample ( $m_b = qm$ )
q	- Number of decisions (or words) per voice sample
m	- Number of bits per decision
$\mu$	- FM index of modulation
"knee"	- That value of signal-to-noise ratio for which the decision error noise = quantization noise
$N_o$	- Noise power density in watts/cps
U	- Utility function

PN-FM	- Analog frequency modulation of a subcarrier using pseudo-noise multiplexing
f-t Matrixing-	Generic for communications techniques using discrete frequency and time multiplexing
M-ary	- Refers to an alphabet of order M
AGC	- Automatic gain control
TWT	- Traveling wave tube
DSBSC	- Double sideband suppressed carrier
$\alpha$	- Error rate at the output of the detector
$\eta^2$	- Average signal-to-noise ratio at the output of the matched filter
$Z(t)$	- Complex signal
$n(t)$	- Complex white gaussian process of spectral density $N_0$ watts/cps
A	- Waveform amplitude
$\omega_0$	- Carrier frequency
$\phi(t)$	- Pseudo random phase modulation
$\tau_p$	- Relative time shift of the $p^{\text{th}}$ clutter signal
E	- Signal energy
$P_K$	- Power of $k^{\text{th}}$ clutter signal
$Q(X)$	- Multiplexing penalty function in db
X	- Computational function of $P_S/N_K$ and G.
C	- Channel capacity
$C_\infty$	- Infinite bandwidth channel capacity

log - natural logarithm unless otherwise specified

$$\exp_2 X \equiv 2^X$$

Conventional - Binary transmission using non-pseudo noise signals to  
PCM represent a "one" or a "zero" (i.e., on-off, FSK, PSK, etc.). The presence or absence of a "one" or a "zero" is detected by making a decision on each bit.

Signal - A generic term for unique waveforms specifically  
Address assigned to particular users or stations

FMFB - Frequency modulation with feedback

## Section 1

### INTRODUCTION

The goal of this contract is to develop, analyze, and compare modulation techniques for multiple-access satellite communications systems. Results pertaining to development and analysis goals were reported in the Phases I and II report.<sup>(1)</sup> The comparison of modulation techniques is the principal subject of this document.

This program has been restricted to the study of those modulation techniques of particular interest for voice communications using active synchronous satellite systems. The study has considered the complete spectrum of modulation techniques to ensure a final comparative analysis of the most promising candidate techniques. In many cases, the analysis was extended to include the specification of system configurations and parameters, where necessary, for a meaningful evaluation of the modulation techniques.

By its very nature, a quantitative comparative analysis requires that some standard or reference system be specified. The reference system most commonly used today is the SSB-FDM telephone system. This system is used as the reference throughout this report and serves as the basis for two types of comparisons:

- (a) Comparison based on communications channel parameters.
- (b) Comparison based on the operational flexibility and implementation complexity of various modulation techniques.

The fundamental methods of comparative analysis used in this report have been studied in detail in Ref. (2). It is assumed that the reader is familiar with this work, summarized in Appendix D. These methods have been extended to include pseudo-noise (PN) techniques. In particular, the theory is extended to digital communications using higher-order (M-ary) signal alphabets with PN-multiplexing, as well as to analog modulation systems multiplexed by means of PN-

subcarriers. Basic questions as to the theoretical performance of PN-multiplexing techniques are answered in this report.

Spread spectrum modulation methods known as F-T Matrixing likewise have been evaluated. It was found that the theoretical performance for high quality voice is much poorer than that of the PN and conventional methods. The analysis is given in Appendix 8.

The key performance parameters in the analysis are the test tone-to-noise ratio (T. T. /N), the full load sinusoid (intrinsic) signal-to-noise ratio ( $P_S/N_K$ ), and channels (i. e., talkers) per megacycle (G). These parameters, their origins and significances are discussed in the appendixes to this report.

The theoretical study of modulation techniques orders the systems along two scales for a given voice channel quality (test tone-to-noise ratio). The scales are:

(a) Intrinsic signal-to-noise ratio in a conventional telephone channel  $0 < (P_S/N_K)_{DB}$

(b) Number of voice channels per megacycle of RF

bandwidth  $0 < G \leq 250$

The significance of the parameter  $(P_S / N_K)$  can be seen by noting that for SSB-FDM, the  $(T. T. / N)$  is directly proportional to  $(P_S / N_K)$ . Therefore, one can conveniently relate the results to the selected reference system. (This approach has been commonly used to compare SSB with wideband FM.)

The theory developed here uses approximations which are good for voice systems of reasonable and excellent quality. However, whereas a large amount of experimental data is available for conventional systems, data is lacking for the PN-systems. A recommended step beyond the work reported here is further study by means of experimentation over a SYNCOM channel.

The operational and implementation part of the study compares the various techniques on the basis of on-board hardware complexity, ease of channel assignment, ground station hardware complexity, and system flexibility. The comparison here will be given by means of tables.

This final report is divided into eight sections. The mathematical results used for the comparative analyses are discussed in Section 2. These results are derived in detail in the appendixes. Based on these results, several measures of comparison of modulation techniques are discussed in Section 3, using graphical data prepared from the theoretical equations as a source. In Section 4, comparisons are made of the modulation techniques, while the system parameters are compared in Section 5. Conclusions, performance improving operations, the strong talkers problems, and recommendations are discussed in Sections 6 through 9 respectively.

The principal conclusions as stated in Section 6 are:

- (a) For high-quality voice corresponding to  $T.T./N = 48\text{db}$ , SSB-up and composite FMFB down, and PN multiplexing with higher-order (M-ary) alphabets are approximately equivalent with respect to talkers per megacycle and  $P_S/N_K$ .
- (b) In the case of lower quality voice ( $T.T./N = 42\text{db}$ , for example) the power requirements specified by

$(P_S/N_K)_{DB}$  for FM and PN are approximately the same. However, from a channel bandwidth utilization point of view, FM is more efficient.

- (c) PN requires less complexity of satellite equipment in comparison to SSB up and composite FM down.
- (d) Where RF bandwidth is not a severe constraint, PN-multiplexing techniques require the least amount of  $P_S/N_O$  in the down link. \*

As a result of conclusions (a) and (d), it is recommended that further work be performed primarily in the experimental area. Therefore, IBM's principal recommendation is that a comprehensive experimental program using RANSAC over a SYNCOM channel be initiated. This recommendation is discussed in detail in Section 9.

\*  $P_S/N_O$  is the ratio of the equivalent sinusoidal power referred to the ground receiver to the receiver noise spectral density (watts/cps).

## Section 2

### MATHEMATICAL RESULTS USED IN COMPARISON STUDY

The mathematical results discussed in this section are developed in greater detail in the appendixes. Appendixes A and B develop the theory of PN-multiplexing using digital and analog modulation techniques respectively, while Appendix C summarizes the theory of conventional multiplexing as developed by Stewart and Huber.<sup>(2)</sup>

Subsections 2.1 and 2.2 summarize the mathematical concepts needed to perform the comparisons made in later sections and to give the reader a feel for the significance of the important equations.

#### 2.1 Conventional Modulation Techniques

The comparison method of particular interest in this study is the procedure used in Reference 2 (and by others) in which the single voice channel characteristics are specified in terms of the test tone-to-noise ratio at the point of zero relative level located at the toll switchboard. This work has been developed for conventional modulation techniques and is extended in this study to include PN-multiplexing.

This criterion is widely used in wire-line carrier systems, and requires definition. To specify the power of a signal in a voice channel at some point in a wire transmission system, it is convenient to measure the power of the signal (in db's) at that point as a multiple of the power that exists at some reference point under test conditions. The reference power is referred to as the zero relative level, and is developed at the test point located at the toll switchboard. When a test condition is developed by applying a 1000 cps sinusoidal voltage (or test tone), this signal develops 1 mw of power at the point of zero relative level. With this method, the absolute power level of  $x$  db mw developed along the link corresponds to a relative level of  $x$  db.

In using the test tone as a voice channel reference signal, and considering the test tone-to-noise ratio at the zero relative point, it is convenient to assume the voice bandwidth to be 3.1 kc. This voice bandwidth combined with the frequency response characteristic of the human ear, or psophometric weighting, provides a noise reduction of 3.5 db over a white gaussian noise band of 4 kc. The comparisons of Stewart and Huber take

(2)

advantage of this noise improvement, and the same benefits will be assumed in this study where appropriate.

The modulation techniques have been compared <sup>(2)</sup> by using the SSB-FDM voice channels as a reference. This channel has been investigated extensively, both analytically and experimentally, so it is no accident that the SSB-FDM channel is used as a reference. The combined voice signal in the SSB-FDM channel is a fluctuating signal, whose statistical characteristics are discussed in Section 7.

Briefly, the instantaneous amplitude of the multiplexed voice signals can be represented in terms of a model, which is a random signal with gaussian amplitude distribution. This model becomes more accurate as the number of component voice signals increases.

The peak power at the point of zero reference level of the multiplexed voice signal is defined as the power level exceeded by the gaussian model no more than 0.003% of the time.

A multiplex full load sinusoid is defined as a sinusoid whose peak power at the zero relative level is equal to the peak power of the multiplexed voice signal just defined.  $P_S$  is defined as the average power of the multiplex full load sinusoid measured at

the zero relative level.

The modulation comparison is performed by specifying the voice channel quality in terms of the test tone-to-noise ratio, and determining the input full load sinusoid-to-noise ratio,  $P_S/N_K$ , that gives rise to the required test tone-to-noise ratio for various modulation techniques. In this case,  $N_K$  is the total thermal noise in the bandwidth occupied by K, SSB-FDM channels.

Appendix D summarizes the principal results of Reference 2.

(Note that  $P_S/N_K$  does not have the physical significance in the case of frequency modulation, for example, as it does in the SSB-FDM case.)

The effect of intermodulation noise, if it exists, has not been incorporated in the results summarized in Appendix D, with the exception of the cases for Frequency Division Multiplex Phase Modulation and Multicarrier PCM. The amplifiers (in particular, the satellite repeater amplifier) have a nonlinear power-out versus power-in relation, so the amplifier operating point and the input signal dynamic range must be constrained so as to restrict the intermodulation noise within specified bounds.

The noise power ratio (NPR) specifies the level of the noise due to

intermodulation in the voice channel. This quantity is defined as the signal-to-noise ratio when the input to the SSB-FDM channel under consideration is white noise with average noise power equal to CCIR's equivalent noise power for SSB-FDM voice channels. (2, 15) The signal power is the average power measured in a specific voice channel under the conditions specified. The noise power is the average power measured when the input to the voice channel under consideration is made equal to zero, but the other channels have the same input applied as in the signal power measurement case.

For the SSB-FDM case, in order to meet a noise power ratio requirement of 34.5 db, a test tone-to-noise penalty of 2.7 db must be met. This assumes that the square root of the instantaneous power output versus the corresponding input quantity is a third-order polynomial; also, that the total input power at any given time is allowed to exceed the input power level that gives rise to the maximum output power,  $P_{\max}$ , only 0.01 % of the time.

In the case of Frequency Division Multiplex Phase Modulation, the case of 1% overload and 0.1% are considered in Reference 2.

Hence, equation (D-6a) includes the effect of intermodulation noise.

In applying the results of the FM performance <sup>(2)</sup> to the SSB up and composite FM down case, it will be assumed that in the combining of SSB to composite FM at the satellite the 3 db test tone-to-noise penalty of SSB is incurred.

## 2.2 PN - Multiplexing Techniques

### 2.2.1 Digital Modulation

Appendix A develops in some detail the theory of PN multiplexing using digital modulation. This section summarizes this work and gives several examples to illustrate its significance.

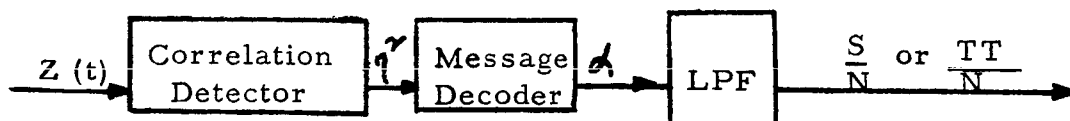


Figure 1. Receiver

Figure 1 is a block diagram of a typical receiver. The correlation detector can be either RF phase coherent or RF phase incoherent. It can be shown that for a higher order alphabet with the order of the alphabet  $M \gg 1$ , there is little difference between coherent and incoherent reception. The analysis used here will assume the incoherent (matched filter followed by

an envelope detector) case.

The input signal  $Z(t)$  includes the  $m^{\text{th}}$  message waveform  $Z_m(t)$ , the clutter signals  $Z_c(t)$ , and thermal noise  $n(t)$ : (see addendum)

$$Z(t) = AZ_m(t) + Z_c(t) + n(t) \quad (2-1)$$

It is assumed that at the particular receiver of interest,  $m$ , there are  $M$  equiprobable signaling waveforms of which only one is sent at a time. This particular signaling waveform combined with thermal noise and the sum of the clutter signals is present at the input to the matched filter. If the input signal matches a particular filter, the average signal-to-noise ratio at the output of the matched filter at the instant of match will be

$$\eta^2 = \left( \frac{P_S}{dN_K} \right) \frac{W_o T}{\left( \frac{P_S}{N_K} \right) \left( \frac{W_o}{2W} \right) (K-1) + 1} \quad (2-2)$$

as derived in Appendix A.

$\frac{P_S}{N_K}$  = full load signal-to-noise ratio (intrinsic signal-to-noise ratio)

$K$  = number of voice channels

$T$  = duration of signal waveform

$2W$  = RF bandwidth

$W_o$  = audio bandwidth (i. e. , 4 kc)

$d$  = duty factor (or activity factor)

The matched filter block actually consists of M matching operations. Therefore, there is an output for each of the M possible signals. It is the function of the decoder to make a decision as to which signal is received. Assuming orthogonal signals, a maximum likelihood incoherent detector, and greatest - of decision, the error rate is related to the output signal-to-noise ratio by the expression:

$$\alpha = \frac{M-1}{2} \exp \left\{ -1/2 \eta^2 \right\} \quad (2-3)$$

The large signal-to-noise assumption is reasonable since we are concerned with the case of high quality speech. For the case of biphase message coherent modulation there is a 3 db advantage over orthogonal (M=2) signals, since a correlation coefficient of -1 can be obtained in the former.

The audio signal-to-noise ratio at the output of the low pass filter is shown in Appendix A to be:

$$\left( \frac{S}{N} \right) = \frac{1}{2\alpha + Q(M)} \quad (2-4)$$

where Q(M) is the quantization noise. This fidelity criterion is a good approximation to both binary and M-ary voice transmissions and is used here for these cases.

In general,  $Q(M) = 2^{-2m_b}$  where  $m_b$  is the total number of bits in the sample waveform. If  $q$  decisions of  $m$  bits each are made,

$$m_b = qm \quad (2-5)$$

Therefore,

$$Q(M) = M^{-2q} \quad (2-6)$$

To complete the analysis one must relate  $(S/N)$  to  $(T.T./N)$ , which is also discussed in Appendix A:

$$(T.T./N)_{DB} = (S/N)_{DB}^{-6} \quad (2-7)$$

By combining Equations (2-2) to (2-7) one can express  $(T.T./N)$  as a function of the communication channel parameters:

$$(T.T./N) = f\left(P_S / (dN_K), G, M, q\right) \quad (2-8)$$

where  $K \gg 1$ , and,

$$G = 250 \left[ \frac{W_o K}{2W} \right] = \text{channel/mc}$$

Eq(2-8) is given by Equation (A-34) of Appendix A and is included here:

$$\begin{aligned} (T.T./N)_{\text{total}} = 2.2 \left( \frac{P_S}{dN_K} \right) & \frac{W_o T}{\frac{P_S}{N_K} \frac{W_o}{2W} (K-1)+1} - 10 \log_{10} (M-1) \\ & - 10 \log_{10} \left\{ 1 + \frac{Q(M)}{M-1} \exp \left[ 1/2 \left( \frac{P_S}{dN_K} \right) \cdot \right. \right. \\ & \left. \left. \frac{W_o T}{\frac{P_S}{N_K} \frac{W_o}{2W} (K-1)+1} \right] \right\} - 6 \end{aligned} \quad (2-9)$$

The function  $G$  gives the number of channels per megacycle. It is also a measure of the mutual interference and vanishes when the RF bandwidth becomes extremely large, as required. This does not mean, however, that the channel capacity vanishes. On the contrary, the channel capacity becomes  $\frac{P_S}{N_0}$ , which is thermal noise limited. In fact, when the bandwidth is infinite the maximum amount of information can be transferred since there is a complete absence of clutter. When the power is infinite the amount of information per active user which can be transferred is clutter limited.

In summary, the expression for the test tone-to-noise  $(T.T./N)_{DB}$ , equation (2-9), contains the effects of thermal noise, clutter, and quantization noise. This expression is general in the sense that it is applicable to conventional binary systems as well as for the  $M$ -ary alphabet. For  $M \gg 1$ , it includes the case of coherent detection. The improvement achieved by using biphase modulation can be included by recognizing that a 3 db improvement will be realized in  $\frac{P_S}{N_K}$ . The proper interpretation of  $d$  (i. e.,  $d = 1/2$ ) enables one to consider on-off binary as well. The time duration per decision is

always made less than or equal to the sampling rate.

To further interpret the results of equation (2-9), consideration will be given to the case in which the decision error is equal to the quantization noise. This condition will be referred to loosely as operating at the "knee" of the  $(T.T./N)_{DB}$  vs  $P_S/N_K$  curve. In the binary case this occurs approximately when

$$2\alpha = \frac{1}{M^2} \quad (2-10)$$

or in the general M-ary case the approximate location of the knee occurs when,

$$2\alpha = M^{-2q} \quad (2-11)$$

Using equation A-36, A-41, we obtain  $\left(\frac{T.T.}{N}\right)$  at the knee

$$\left(\frac{T.T.}{N}\right)_{DB} = 3(2mq-3) \quad (2-12)$$

The parameter  $q$  is the number of words per voice sample and  $m = \log_2 M$  is the number of bits per word. For example,  $m=1$ ,  $q=10$  represents conventional PCM, transmitting 10 bits per sample and requiring 10 binary decisions per sample (i.e., 10 one-bit words). From the test tone-to-noise ratio,  $(T.T./N)_{DB}$ , at the knee of the  $(T.T./N)_{DB}$  vs  $P_S/N_K$  curve, it is clear that the same voice quality can be obtained in the

cases of  $m=1$ ,  $q=10$  and  $m=10$ ,  $q=1$ . However, in the latter case single-word decisions are made, and 1024 waveforms are required for each channel, one for each sample value. This represents an extremely complex system unless the waveforms used are delayed versions of the same signal, for example, pulse rate modulation (or equivalently digital PPM) .

To further comprehend the system operation at the knee of the curve,  $W$  will be made very large in equation (A-36) and solved for  $P_S/N_K$ . This value is denoted as  $(P_S/N_K)_\infty$  and is given in equation (A-43)

$$\left( \frac{P_S}{N_K} \right)_\infty = 4qd \left[ 2q \log M + \log (M-1) \right] \quad (2-13)$$

This is an extremely important expression and represents the threshold (intrinsic) signal-to-noise ratio for the case where the RF bandwidth is much greater than  $\frac{P_S}{N_0}$  so as to permit the effect of clutter to be neglected. When clutter can be neglected, in the bit-by-bit decision case,  $M = 2$  , we have:

$$\left( \frac{P_S}{dN_K} \right)_\infty = 8q^2 \log 2 \quad (2-14)$$

For the same case, in the word-by-word decision case,  $q=1$ ,  $M \gg 1$ ,

$$\left( \frac{P_S}{d N_K} \right)_{\infty} = 12 \log M \quad (2-15)$$

Thus, from Equation (2-14), when  $q=10$ , we have 27.2 db.

$\left( \frac{P_S}{d N_K} \right)_{\infty} = 19.2$  db is obtained from Equation (2-15) for the same number of message bits. The higher-order alphabet therefore requires approximately 8 db less power than a conventional orthogonal binary system for good voice quality. However, the less efficient system is somewhat simpler. If on-off binary is used, a 3 db improvement in  $\left( \frac{P_S}{N_K} \right)$  is achieved since  $d=1/2$ .

Similarly, if biphase modulation coherent detection is used a 3 db improvement is also achieved. For large-size alphabets there is little difference in performance between coherent and incoherent reception.

As a possible compromise between complexity and efficiency, we can assume that  $q=2$ ,  $m=5$ . This gives the same quality and reduces the complexity to a 32-level alphabet. Now, two word decisions per sample are required with a reduction

in efficiency. The 32-level alphabet system is a practical one even for complex signal alphabets. For this case, from equation (2-13),  $\left(\frac{P_S}{dN_K}\right)_\infty = 21.5 \text{ db}$ . This results in a 2.3 db increase in power over the more complex M-ary system and a savings in complexity by a factor of 32. On the other hand this two-word per sample system is 6 db more efficient than orthogonal binary but only 3 db more efficient than on-off or biphase modulation. The reduction in system complexity of conventional binary over the two-word system is now reduced further.

### 2.2.2 PN-FM Modulation

In the FM case the voice signal modulates a conventional FM subcarrier which in turn is modulated by the PN subcarrier. The FM signal is extracted the same way except that after removing the PN-subcarrier at IF the signal is fed into a conventional FMFB receiver. For FMFB we will use a model developed in reference 3. Here it is shown that a valid model for FMFB is to assume an M-ary alphabet with a greatest - of decision for determining the channel which contains the received frequency. The analog measurement of the frequency is then obtained by feeding this channel output into a frequency

discriminator. It has been shown that the model agrees with FMFB required results. <sup>(3)</sup> Clearly, such a model is primarily of theoretical interest since it permits analytical treatment of FMFB. In this report, in particular, the theory of M-ary PN systems, which have been developed for digital transmission, is now directly applicable to FMFB; this theory is shown in Appendix B. The order of the alphabet is directly proportional to the FM index.

For the purpose of this discussion assume that the PN-signal is spread over an extremely large band so that the mutual interference can be neglected. Using equation (C-7) we have

$$\left( \frac{T.T.}{N} \right)_{FM} \propto = 10 \log_{10} \left( \frac{P_S}{dN_K} \right) + 20 \log_{10} \mu - 4 \quad (2-16)$$

This is clearly the expression for a conventional FM system above threshold.

In the presence of mutual interference, the mathematical expressions are more complex and require a slight modification of the basic theory. This is also developed in Appendix B.

### 2.2.3 PN-Multiplexing Loss

The function  $Q$ , defined in equation A-46a, specifies the additional db which must be added to  $(P_S/N_K)_\infty$  to overcome the clutter in order to maintain the same voice quality as in the absence of clutter. This function is defined at the knee of the  $(T.T./N)$  versus  $(P_S/N_K)$  curve. From Eq A-50,

$$Q = 10 \log_{10} \left[ \frac{250}{250 - G (P_S/N_K)_\infty} \right] \quad (2-17)$$

### 2.2.4 Utility Function For Optimizing "Power-Bandwidth" Product

The parameters  $P_S/N_K$  and  $G$  vary over a wide range. To select these parameters in some optimum sense, one needs to define a criterion. An interesting utility function is given in Eq A-57,

$$U = \left( \frac{P_S}{N_K} \right)_{\text{db}} \frac{1}{G} \quad (2-18)$$

Since  $(P_S/N_K)_{\text{db}}$  is proportional to satellite power in db and  $1/G$  is proportional to bandwidth, optimizing  $U$  gives a minimum power-bandwidth product.

Where  $\left( \frac{P_S}{N_o} \right)$  in the down-link is expensive and not RF bandwidth, the system should be designed so that performance is thermal noise limited (i.e.  $2W \geq 3 \frac{P_S}{N_o}$ ). The intrinsic signal to noise ratio will then approximate  $\left( \frac{P_S}{N_K} \right)_\infty$  and  $G \approx 0$ . Figure 12 shows curves of  $\frac{P_S}{N_K}$  vs.  $G$  for M-ary alphabets in this region of interest.

### Section 3

#### THEORETICAL PERFORMANCE CURVES DISCUSSION

We have graphed a variety of characteristics which show the behavior of PN-multiplexing systems. We also show a family of curves of the theoretical output signal-to-noise ratio  $(S/N)_o$  for SSB, FM, and PCM (for M-ary coherent detection), Figures 9 and 10. The FM curves are shown with and without feedback, demonstrating the threshold effect. These curves are obtained from Ref 3. The dashed curves represent the characteristic in the threshold region which would be obtained if FMFB is used, while the solid curves represent the FM characteristic without feedback. The PCM curves are for M-ary orthogonal signals and demonstrate the PCM threshold.

Curves showing the intrinsic signal-to-noise ratio  $(P_S/N_K)$ , as functions of the number of channels per megacycle, are also presented. In PN-FM it is assumed that the FM-bandwidth is not greater than the PN subcarrier band.

All the curves shown assume an activity factor of 25%. In case of on-off binary the activity factor is only 12.5% while in biphase binary the activity factor is 25%. However, since the

biphase binary symbols are negatively correlated, a 3 db improvement over orthogonal binary is obtained making on-off and biphase equivalent as far as PN multiplexing is concerned

The number of channels per megacycle is inversely proportional to the activity factor as shown in equation (A-53) (and to the correlation coefficient in biphase modulation). Similarly, the intrinsic signal-to-noise ratio  $(P_S/N_K)_{db}$  is reduced by the logarithm of the activity factor. Thus, PN-multiplexing systems should take full advantage of the activity factor.

The curves which are shown here are obtained by using the equations in the Appendixes.

### 3.1 Conventional Modulation Techniques

The curves shown in Figure 9 are  $(S/N)_o$  vs  $P_S/N_K$  for various FM indexes with and without feedback. (These curves are not  $(T. T. / N)$  since appropriate loading characteristics have not been included.) These curves are obtained from Ref 3. The dashed curves represent the characteristic in the neighborhood of the threshold, while the solid lines are those without feedback. It is evident that the threshold signal-to-noise ratio without feedback increases at a greater rate with increase in the

modulation index than FM with feedback. A curve showing this behavior more precisely for FMFB is Figure 11. This curve was obtained from equation (B-9). The envelope of the FMFB threshold characteristic (i. e., the knees) is similar to that obtained in M-ary PCM since the mathematical model of FMFB which is used in Ref 3 postulates an M-ary orthogonal decision procedure as a part of the FM reception process. In the threshold region the M-ary error probability determines the characteristic.

Curves of  $(T. T. / N)$  vs  $(P_S / N_K)$  for conventional systems are shown in Ref 2.

### 3.2 PN Multiplexing

As discussed previously, PN multiplexing includes digital message transmission and analog message transmission. In Ref 3 the intimate relationship between FMFB and digital transmission using M-ary sinusoidal signal alphabets is shown. The computation procedure for PN multiplexing using both digital and analog transmission is discussed in Appendix C. This procedure was used to obtain the curves which will be discussed here.

### 3.2.1 Digital Transmission

Figure 1 is a curve of  $\left(\frac{T.T.}{N}\right)$  in db vs  $q$ , the number of words per message sample. The test tone-to-total noise ratio (includes decision plus quantization noise) is computed at the knee of the PCM threshold characteristic.

It is a graph of equation (C-1).

Figure 2 is a curve of  $\left(\frac{P_S}{N_K}\right)_\infty$  vs  $q$ , equation (C-2). This quantity,  $(P_S / N_K)_\infty$ , is obtained by equating decision noise to quantization noise and letting the RF bandwidth  $W$  become infinite.

From Figures 1 and 2, we obtain  $\left(\frac{T.T.}{N}\right)_\infty$  vs.

$\left(P_S / N_K\right)_\infty$  of Figure 3 when the PN signal bandwidth is made infinite. The curves shown are the envelopes of the knees of the threshold characteristics. An increase in  $\left(P_S / N_K\right)_\infty$  by 3 db will increase  $\left(\frac{T.T.}{N}\right)_\infty$  by 3 db at which point performance is quantization noise limited. As an example for a 45 db  $(T.T./N)$ , a 9-bit M-ary alphabet requires an intrinsic signal-to-noise ratio,  $(P_S / N_K)_\infty$  of 12.8 db. A three-word per sample signal alphabet ( $q = 3$ ) requires

$(P_S/N_K)_\infty$  of 16.4 db for the same quality. On the other hand, a bit-by-bit orthogonal decision procedure requires a  $(P_S/N_K)_\infty$  of 20.6 db. On-off and biphase modulation requires only 17.6 db. Thus for a given quality, the M-ary alphabet gives the most efficient performance as far as intrinsic signal-to-noise ratio is concerned.

In order to calculate the intrinsic signal-to-noise ratio as a function of the number of channels per megacycle (i. e., for finite bandwidth), we first obtain the function  $Q(X)$  shown in Figure 4.  $Q(X)$  is the multiplexing penalty in db which must be paid due to a finite RF bandwidth. The number of channels per megacycle is then related to  $X$  and  $(P_S/N_K)_\infty$  (not in db) via equation (C-5).

Figure 5 shows  $(P_S/N_K)_\text{db}$  vs  $G$ , the number of channels per megacycle, for  $(T.T./N) = 39$  db and 45 db. It is clear that the M-ary alphabets ( $q = 1$ ) are most efficient in power and bandwidth. The least efficient digital technique is orthogonal binary ( $m = 1$ ). Voice quality of 39 db can be achieved with an intrinsic signal-to-noise ratio of 17 db and 10 channels per megacycle. The latter assumed an activity factor of 25%.

A unity activity factor will yield only 2.5 channels per megacycle and will require 6 db more power.

Figure 6 compares the M-ary alphabet to bit-by-bit decisions using orthogonal signals, on-off binary and biphasic binary. Clearly, the higher-order alphabet is more efficient in power and bandwidth. This result is expected when it is recognized that the M-ary alphabet is equally effective against thermal noise and clutter. Thus, both thermal noise and clutter rejection can be achieved simultaneously, by increasing the alphabet order.

Figure 7 shows a family of curves which represents a utility function for choosing an operating point based on efficient use of both power and bandwidth. A flat minimum exists. Where bandwidth efficiency is not required this function loses much of its usefulness.

Figure 6 shows curves of  $(P_S/N_K)$  vs  $G$  for pseudo-noise multiplexing with FMFB. For the FM indexes chosen, the curves are steeper than those for digital transmission. Performance is far superior to bit-by-bit decision binary communication but substantially inferior to M-ary digital systems. The utility function for these systems is shown in Figure 8, and is also far inferior to M-ary digital systems. (FMFB is shown in Ref 3 to be a type of M-ary orthogonal alphabet.)

Other forms of analog modulation using PN multiplexing will exhibit a similar behavior if the modulation exchanges bandwidth for signal-to-noise.

### 3.3 General Conclusions About PN Multiplexing

\*For good quality voice transmission PN multiplexing systems require the use of M-ary alphabets in order to achieve low values of intrinsic signal-to-noise ratio ( $P_S/N_K$ ) and relatively high values of channels per megacycle. The M-ary alphabets are equally effective against thermal noise and clutter.

\*Efficient utilization of power and bandwidth requires full exploitation of the activity factor, since the power required is directly proportional to it and the channels per megacycle is inversely proportional to this factor.

\*Where RF bandwidth is not of primary concern but  $\left(\frac{P_S}{N_o}\right)$  is, then optimum use of such a channel requires that  $2W \geq 3 \frac{P_S}{N_o}$  i. e.

operation should be thermal noise limited.

\*When  $2W \gg \frac{P_S}{N_o}$  performance is thermal noise limited and when  $2W \ll \frac{P_S}{N_o}$  performance is clutter limited.

\*The PN signal is a "sub-carrier" which rejects interference by other PN subcarriers that use the common channel.

## Section 4

### COMPARISON OF MODULATION TECHNIQUES

In this section we will compare the modulation techniques for a given test tone-to-total noise ratio. In the conventional analog systems we will assume that the test tone-to-thermal noise ratio is equal to the test tone-to-distortion ratio. Thus, if we compute the test tone-to-thermal noise ratio we will subtract 3 db to obtain the test tone-to-total noise ratio. In PN multiplexing the computations automatically include the total distortion.

We will compare modulation techniques using PN multiplexing, various modulation techniques using conventional multiplexing, and finally conventional multiplexing with PN multiplexing. Two parameters will be compared: the intrinsic signal-to-noise ratio and the number of channels per megacycle of RF bandwidth. The values of these parameters will be determined for a given quality of performance specified by the test tone-to-total noise ratio.

In all cases the choice of modulation parameters will reflect practical considerations.

For those techniques using digital transmission we will operate 3 db above the knee of the operating characteristic. Thus,  $(P_S/N_K)$  will be increased by 3 db. We will also increase the value of  $(T.T./N)$  by 3 db so that the acceptable range of  $(T.T./N)$  will actually be between 45 and 48 db.

We will assume a 25% activity factor in all PN systems recognizing that in the correlation-locked techniques (1) on-off operation is more difficult to achieve. The results obtained assume that RF phase information is not used although in the correlation-locked techniques this type of operation will improve performance. Our results will therefore be conservative in this case. For good voice quality, performance is only slightly improved when RF phase-lock is used, except in the special case of biphase modulation. Here, we add a 3 db improvement in the intrinsic signal-to-noise ratio and double the channels per megacycle.

All systems will be classified into three categories:

- (1) Very good quality, T. T./N = 48 db
- (2) Good quality, T. T./N = 42 db
- (3) Acceptable quality, T. T./N = 36 db

A table will be used for this comparison. we will require at least two channels per megacycle. Any modulation technique which cannot satisfy this will be automatically eliminated. Thus, a 200-channel system will require a satellite bandwidth less than 100 mcps.

#### 4.1 Comparison of PN-Multiplexing Techniques

The comparison here is shown in Table 1. The two best PN modulation techniques are: M-ary digital message transmission and PN-FMFB. (The latter is an analog form of M-ary transmission with M small.) The choice of the optimum modulation technique is based on the minimum value of the utility function. It is quite clear that the M-ary systems are far more efficient than PN-FM for the indicated parameters. For acceptable voice

quality ( $T. T./N = 36$  db), PN-FMFB is competitive, particularly where power and not bandwidth is at a premium, as is the case in satellite communication. Here, however, on-off binary is almost as good and much simpler to implement. Thus, based on implementation, on-off binary is preferable to PN-FMFB.

Table 2 shows a table of the M-ary system with a utility function displaced from the minimum so as to reduce satellite power. This tradeoff causes only a slight decrease in the number of channels per megacycle. The utility function is also increased only slightly. Based on the utility function performance is suboptimum, although still substantially better than PN-FMFB. These parameters are perhaps more useful than those based exactly on the minimum value of the utility function, since less satellite power is required here with only a small loss in the channels per megacycle. These results will be compared with FMFB using conventional multiplexing.

#### 4.2 Comparison of Conventional Multiplexing Techniques

Table 3 shows a comparison of three conventional multiplexing techniques based on the intrinsic signal-to-noise ratio and the number of channels per megacycle. Clearly, SSB has the maximum number of channels per megacycle of any modulation system but it also requires substantially more power than FM-FB and on-off conventional PCM. However, FMFB is superior to PCM in satellite power requirements. In the region of medium-to-acceptable voice quality FMFB also obtains more channels per megacycle than PCM. Where bandwidth is at a premium and the power

Table 1. Comparison of PN Multiplexing Techniques  
(PN, M-ary with PN-FMFB)

PN-System	$(T.T./N)_{DB}$	$(P_S/N_K)_{DB}$	G	U
m = 9, q = 1	48	23.0	10.8	2.13
FM, $\mu = 25$	48	23.4	2.58	9.05
m = 8, q = 1	42	22.5	12.3	1.83
FM, $\mu = 16$	42	21.4	4.0	5.30
m = 7, q = 1	36	22.0	14.0	1.57
FM, $\mu = 10$	36	19.4	6.3	3.08

Table 2. M-ary, PN-Multiplexing Parameters

M-ary PN System	$(T.T./N)_{DB}$	$(P_S/N_K)_{DB}$	G	U
m = 9, q = 1	48	21	9.4	2.24
m = 8, q = 1	42	21	11.2	1.88
m = 7, q = 1	36	20	12.2	1.64

Table 3. Comparison of Conventional Modulation Techniques

Mod. Tech	$(T, T/N)_{DB}$	$(P_S/N_K)_{DB}$	G
SSB	48	42	250
$\mu = 10$ , Comp. FMFB	48	20	11.4
$q = 9$ ; PCM (on-off)	48	26.6	13.9
SSB	42	36	250
$\mu = 4$ ; Comp. FMFB	42	22	25
$q = 8$ ; PCM (on-off)	42	25.6	15.6
SSB	36	30	250
$\mu = 2$ ; Comp. FMFB	36	22	41.6
$q = 7$ ; PCM (on-off)	36	24.4	17.8

Table 4. Performance Parameters—PN Multiplexing and Composite FMFB

Mod. Tech	$(T, T/N)_{DB}$	$(P_S/N_K)_{DB}$	G	U
$m = 9, q = 1$	48	21	9.4	2.24
FMFB; $\mu = 10$	48	20	11.4	1.75
$m = 8, q = 1$	42	21	11.2	1.88
FMFB; $\mu = 4$	42	22	25.0	0.88
$m = 7, q = 1$	36	20	12.2	1.64
FMFB; $\mu = 2$	36	22	41.6	0.53

constraint is somewhat relaxed, SSB, or narrow deviation FM ( $\mu = 1$ ) are reasonable modulation techniques for acceptable voice quality. For high quality systems it is necessary to exchange channels per megacycle for on-board power.

#### 4.3 PN-Multiplexing Compared to Conventional

Table 4 shows the performance characteristic between the PN, M-ary system and conventional multiplexing using FMFB.

Table 4 shows that for very good quality PN multiplexing and large index FMFB ( $\mu = 10$ ) are comparable as far as the channel parameters are concerned. However, for good and acceptable quality, FMFB using relatively narrow deviation FM makes better use of the channel bandwidth. It is therefore quite clear that PN multiplexing is competitive with FMFB where toll quality performance is required. Where satellite power is at a premium and not bandwidth, the PN systems remain competitive even at reduced quality. In fact, when  $W \gg P_S/N_0$  the PN, M-ary digital techniques are most efficient in the use of on-board power. The utility factor which is useful for making a choice among PN systems is not very useful for comparing PN systems to conventional ones, since the latter are inherently far more efficient in bandwidth for average quality. The utility factor favors systems which make efficient use of bandwidth since it exchanges bandwidth for power on a db basis.

The numbers which have been chosen in this section for the comparison of conventional and PN multiplexing techniques indicate that the same values of  $P_S/N_K$  and G can be achieved for both FMFB and PN multiplexing

in the high quality case. Where RF bandwidth is more important than the down link  $P_S/N_O$ , these parameter values represent a good compromise. Here, the number of channels per megacycle,  $G$ , is an important comparison criterion. However, where the RF bandwidth is not of primary importance, the number of channels per megacycle is not significant as a comparison criterion. The quantity  $P_S/N_K$  is much more significant since it then becomes the limiting factor on system performance. Under these conditions, PN multiplexing using  $M$ -ary alphabets is superior to FMFB and, in fact, is an optimum modulation technique. Whereas conventional FMFB requires an RF bandwidth which is several times less than  $P_S/N_O$ , PN modulation operates most efficiently when the bandwidth is much greater than the noise bandwidth. These conclusions can be illustrated by the following example.

Consider the parameters chosen for  $T.T./N = 36$  db,  $m = 7$  and  $q = 1$ . As shown in Table 2,  $P_S/N_K = 20$  db and  $G = 12.2$ . If  $K = 200$  channels are required, the RF bandwidth will be  $200/12.2 = 16.5$  mcps.  $P_S/N_O$  can be computed as

$$(P_S/N_K) = \frac{P_S}{N_O K W_O} = 100,$$

where,  $W_O = 4$  kc. Therefore  $P_S/N_O = 100 K W_O = 80$  mcps.

For this case, the RF bandwidth is substantially less than  $P_S/N_O$ , yielding a clutter limited system. Here, FMFB will be superior as shown in Table 3.

To contrast this result with a case where PN modulation will be preferred, assume an RF bandwidth of 100 mcps. Again assuming  $K = 200$ , one can compute from equation (A-43) that  $(P_S/N_K) = 14.7$  db. From equation (A-47),  $X = 8.5$  and for all practical purposes  $Q(X) = 0$ . Thus

$$(P_S/N_K) = (P_S/N_K)_\infty$$

and  $P_S/N_O$  becomes  $(P_S/N_O) = (2 \times 10^2) (4 \times 10^3) (29.5) = 24$  mcps.

This results in a power reduction of approximately 5.3 db, at a bandwidth increase by a factor of 6. In order to be of practical value this example must be related to antennae gains, receiver noise figures, etc.

The modulation comparison here is based on theoretical modulation parameters. It now remains to compare these techniques further, this time on operational and equipment considerations. We will now proceed to do this.

## Section 5

### SYSTEM COMPARISON

The comparison based on the theoretical channel parameters represents a partial ordering of the modulation techniques. (In actuality this choice has already been influenced by physical and practical realizability considerations.) It is now necessary to compare the modulation techniques further, using operational and implementation criteria. Four criteria of this type have been selected:

- (1) On-board hardware complexity
- (2) Ease of channel assignment
- (3) Hardware complexity on the ground
- (4) Flexibility

#### 5.1 On-Board Hardware Complexity

It is well known that in a satellite communication system it is desirable to reduce the on-board electronics to a minimum. The system reliability is intimately related to this factor.

One of the major advantages of PN systems is that the on-board electronics takes its simplest form, i.e., a hard limiter (as the multiplexer) or a slow acting AGC. (Frequency translation is, of course, implied.) On the other hand the competitive conventional system, FMFB, requires linear multiplexing of the SSB-up signals at the satellite and then composite

frequency modulation. Thus, whereas the PN signal is made "rugged" at the ground station sending end, the SSB signal must be made "rugged" in the satellite. In addition the up-link must be extremely linear until after the remodulation process takes place. This leads to inefficient utilization of the up-link. The hard limiting of the PN signal, if used, will result in a loss in the effective processing gain which may vary from 1 to 2 db. On the other hand, AGC in the satellite avoids this penalty. The hard limiter loss is reflected as a loss in the number of channels per megacycle (as much as 30%) and also a loss in the intrinsic signal-to-noise ratio. AGC is therefore preferable to hard limiting.

It is well known that bandwidth spreading is an effective modulation technique for reducing interference in conventional narrow-band systems. In addition, the PN systems also have an equal ability to reject interference from conventional systems. Thus the type of spectrum spreading which characterizes PN systems may very well permit the use of extremely broadbands in satellite links to the advantage of both the PN systems and conventional links presently in use.

There is very little doubt that the conventional systems using composite FM will also cause a degree of performance degradation in the modulation process. However, whereas the expected loss due to limiting (if used) can be predicted quite accurately, the losses in the FMFB systems are not as accurately predictable.

In summary, the PN multiplexing techniques require substantially simpler in-the-satellite electronics and are therefore more preferable as far as this criterion is concerned.

## 5.2 Ease of Channel Assignment

A most attractive property of PN systems in particular, and common channel systems in general, is that switching centers are not required for channel assignment. The number of quasi-orthogonal addresses is so large that each channel can be assigned a unique signal address which is sufficiently different from all other addresses. In general, the system performance is independent of the particular structure of the signal addresses in use (for large bandwidth-time products), but depends only on the number which are being transmitted at any one time. Conventional systems, however, are "signal address" limited and hence require channel assignment control either by a monitoring switching center or by some other discipline.

To prevent system overload in a PN-system it is still essential to monitor the clutter environment. A technique for accomplishing this simply but effectively (without the use of a switching center) is described in (1).\*

In summary, PN multiplexing techniques have a decided advantage in that channel assignment is extremely simple. Such a system will not only eliminate a switching center which must monitor worldwide traffic but perhaps more important, the politically sensitive question concerning the country in which to locate the switching center will be nonexistent. The PN method of channel assignment is unambiguous and also superior to those techniques which depend on a certain discipline to which all stations must adhere.

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\*Reference (1) Report pp 4-237

### 5.3 Hardware Complexity on the Ground

The hardware complexity required for conventional systems is better known than that for PN techniques. In addition, empirical data demonstrating the hardware performance is available in greater abundance than in the case of PN systems. This experience with hardware is a decided advantage for conventional multiplexing and is responsible for the use of known modulation techniques on the early experimental satellite links.

The competitive PN techniques require M-ary (higher-order) signal alphabets. This generally implies extremely complex apparatus. However, the PN techniques using pulse rate modulation (PRM) with matched filter reception can achieve extremely large alphabet sizes in delay (1).<sup>\*</sup> This modulation process is no more complex than conventional PRM; in particular, the complexity is for all practical purposes independent of alphabet size. The signal addressing techniques require a maximal length sequence generator with associated digital logic circuitry.

The most complex part of the system is the matched filter required for reception. However, matched filters with, say, 128 taps will suffice; experimental evidence exists that 64 taps will be just as good. In addition, to reduce mutual interference, frequency hopping can be used. Thus, although the actual signal address may contain no more than 64 bits, the effective processing gain against thermal noise and clutter is much greater; for all practical purposes equal to that obtained with CW pseudo-noise multiplexing. Because the modulation is PRM, precise synchronization at the receiver is not required; gating the receiver in the neighborhood of the expected signal will suffice.

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<sup>\*</sup> Reference (1), pp 4-43

Completely asynchronous operation can be achieved if threshold detection is used. This type of operation results in a 2 db loss in the matched filter output signal-to-noise ratio.

Whereas the PN, M-ary systems appear to be extremely complex, upon close examination it is seen that the technique using PRM with matched filter reception turns out to be quite reasonable. With the great progress in integrated circuits, a multi-channel matched filter receiver will not be expensive.

In summary, the experience available with the use of conventional receiving techniques as well as practical knowledge as to design and complexity favors these techniques. FMFB is, however, relatively new and all the answers concerning performance, threshold behavior, tracking errors, etc., are not available. The PN matched filter techniques require some laboratory experimentation although the most pressing requirement is experimentation over real satellite links.

#### 5.4 Flexibility

The chosen PN techniques (1)\* are inherently flexible as far as system expansion is concerned. The number of subscribers can be doubled by adding a single flip-flop to the sequence generator. Similarly, the frequency hopping permits a relatively simple way of expanding the capacity of the system. In fact, the matched filter system lends itself to a modular approach to system design and expansion.

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\*Reference (1), pp 4-198

It is simple to trade off message rate for number of active users. Thus, many more telegraphy channels can be accommodated than voice channels.

Many matched filter channels can be accommodated at a single receiver station simply by adding resistance matrixes with relatively simple post-detection circuitry. Also, many channels can also be multiplexed at a terminal. Here, however, a set of PN sequence generators for addressing the receiving stations may be required.

The PN system proposed with some modification can be given a jamming and spoofing immunity, thus permitting the assignment of channels for military use. In addition it is simple to crypto-secure a channel permitting its use for both diplomatic and military purposes. The inherent privacy of such techniques is well known.

Finally, the PN modulator and demodulator can be viewed as a MODEM which establishes a connection between two terminals much like a wire. At the transmitter several conventional voice channels can pulse rate modulate a signal generator which drives the PN-MODEM or signal address generator. At the ground station, the matched filter receiver demodulates the PRM message and the resulting SSB-FDM signal is fed into conventional telephony equipment.

In the conventional system more subscribers can be accommodated by adding logic to the automated switching center (if used). This is not difficult to do. However, the information handling capability of the FMFB system is complex. This is particularly true since remodulation of the

received SSB-FDM signal is required in the satellite. It is therefore essential to build into the satellite an FM remodulation capability so as to accommodate the maximum anticipated expansion. This, however, is not a particularly flexible situation.

Increasing the number of channels of an FMFB system also requires an extension of the frequency tracking capabilities of the phase-locked portion of the receiver. The difficulty with which this can be accomplished is not clear at this time although it is expected that it increases with the frequency deviation.

In summary, the ability to increase the number of channels of an FMFB system is difficult, particularly since it is constrained by the satellite electronics (i.e., remodulation of the composite signal). PN multiplexing is therefore a more flexible type of modulation.

#### 5.5 Summary of Operational and Implementation Considerations

Table 5 summarizes the comparison based on operational and implementation considerations.

TABLE 5. System Complexity

<u>Modulation Technique</u>	<u>On-board Hardware Complexity</u>	<u>Ease of Channel Assign.</u>	<u>Ground Hardware Complexity</u>	<u>Flexibility</u>
FMFB	Complex	Complex (requires switching center, or other discipline)	Relatively complex	Not flexible
PSEUDO NOISE	Very simple	Very simple	Relatively complex	Very flexible

In this table summary it is seen that PN multiplexing leads to a more flexible satellite communication system, and generally less complex, in terms of on-board electronics.

## Section 6

### CONCLUSIONS

Table 4 indicates that for good quality voice FMFB and PN multiplexing using M-ary alphabets are equally efficient in the use of the communication channel. The breakeven point here is at a  $T.T./N = 48$  db. For lower quality both systems require approximately the same on-board power. However, the FM systems use the bandwidth more efficiently. The utility function of the conventional modulation is from two to three times better than for the PN system.

In conclusion, when on-board power, operational flexibility, and on-board electronic simplicity are important factors then PN multiplexing using pulse-rate modulation (i.e., a digital PPM, M-ary alphabet) is superior to FMFB. However, where bandwidth is the significant factor, then for  $T.T./N = 42$  db, narrow-band FM is superior.

Finally, these conclusions must be tempered by the fact that there exists almost no operational and field experience with modulation techniques of this type, particularly for satellite communication systems. The PN theory developed here is approximate; however, it is expected that the results which will be obtained in practice will not differ from these by a sufficient amount so as to eliminate PN multiplexing from consideration. In short, it is expected that these are valid candidates. Laboratory and field experiments are necessary to determine their precise performance.

## Section 7

### OPERATIONS THAT IMPROVE PERFORMANCE

The problems associated with the transmission of voice signals have received a considerable amount of attention in the telephone industry. These investigations have been concerned with the basic statistics of the single channel voice signal, SSB multiplexed voice channel statistics, methods to specify voice quality, the parameters to specify in designing voice transmission systems that provide the required voice quality, and methods to process voice in analog and pulse code modulated systems to enhance system operation. The interest here is mainly in the last item; however, some of the other basic items will be discussed briefly.

#### 7.1 Characteristics of a Single Voice Channel—General Discussion

The significant parameters that characterize the voice signal of a single speaker are the long term mean power, the instantaneous voltage fluctuation, the activity factor, and the time constants of consonants and syllables. Each of these parameters will be discussed.

The mean power of a voice signal varies from speaker to speaker, but, in addition, in a telephone system the transmission losses of the order of 25 db have been found to exist between a subscriber and the input to a final junction circuit. The quantity referred to as the volume (in db) has been used to measure the mean power of speakers in the telephone plant. A

volume indicator was used to measure this quantity, which is defined as

$$\text{Volume} = 10 \log_{10} \frac{\text{average speech power in milliwatts}}{1.66}$$

The probability density function of the volume has been obtained at the transmitting toll test board, which is the zero transmission level, and has been found to be approximately normal. The mean volume has been measured to be -16 db and the standard deviation as 5.8 db for the particular telephone plant considered. See Table 6.

The volume as defined above is no longer in use, but instead a VU meter is used to determine the power of the speech signal. The calibration of the VU meter is performed by applying a 1 kc signal which dissipates 1 mw in 600  $\Omega$ . With actual speech input the VU meter is read by taking the average of peak deflections about every 10 seconds after discarding the first few high readings. The volume in db can be expressed in terms of VU as follows

$$0 \text{ db} = +6 \text{ VU}$$

The mean and the variance of the volume distribution depends upon talker characteristics and/or equipment improvement as well as other factors. It has been found in recent measurements that the mean volume,  $V_o = -15 \text{ VU}$ , and the standard deviation  $\sigma = 5 \text{ db}$ .

The instantaneous speech voltage distribution has been measured for various fixed volumes and different commercial sets, and it has been found that the ratio of rectified instantaneous speech voltage to rms voltage

TABLE 6

V(db)	$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} \exp \left\{ -\frac{(V - V_o)^2}{2\sigma^2} \right\} dv$
-7.5	0.075
-10.5	0.15
-12.5	0.28
-15.0	0.43
-17.5	0.6
-20.0	0.74
-22.5	0.86

TABLE 7

$E/U = \frac{\text{rectified instantaneous speech voltage}}{\text{rms speech voltage}}$	$\int_{E/U}^{\infty} p(x) dx *$
3.5	.02
2.5	.036
2.0	.062
1.0	0.12
0.75	0.16
0.5	0.22
0.25	0.36
0.125	0.5

\*  $p(x)$  is the density of  $E/U$

is as noted in Table 7. Attempts have been made to approximate analytically the distribution of  $E/U$ . The approximation due to Davenport uses a normal distribution for small values of  $E/U$  and a negative exponential distribution for larger values of  $E/U$ .

The probability that  $E/U$  exceeds 0.125 is given as approximately 0.5. This is an indication that there is a large concentration of small instantaneous speech signal amplitudes smaller than the rms speech voltage. This is partly due to the fact that there are pauses in continuous speech between words and syllables, but the basic problem is that even with speech that is maintained at constant volume the signal intensity varies considerably from syllable to syllable as well as within each syllable. The energy of some consonants is 30 db down in comparison to the stronger vowels. In particular, it has been found that speech spectrum represented in terms of rms sound pressure level vs frequency has a marked peaking between 400 and 800 cps and a steady decline of the sound pressure level at higher frequencies. It will be seen later that the two methods of companding take advantage of these peculiarities of the voice signal.

Some of the more salient characteristics of a single voice signal have been discussed and now the properties of numerous voice signals multiplexed by means of SSB will be considered. The multiplexed voice signals are merely frequency division multiplexed baseband voice channels located at the appropriate portion of the spectrum. In designing an amplifier to process SSB multiplexed voice signals, the statistics of the multiplexed voice signals are necessary. When a total of  $N$  channels are multiplexed

at any given time, there will only be  $n$  active channels. The relation between the instantaneous rectified peak voltage of the multiplexed signal as a function of the number of channels  $N$  is required. This information is used to specify the rms power of a test tone, whose peak power is determined by a level not to be exceeded, a given percentage of the time, by the instantaneous power of the multiplexed voice signal.

In a single voice channel the active time is considered to be the time during which a conversation is taking place allowing for the pauses during ordinary speech. Measurements made on a large group of circuits indicate that during the busiest hour a channel is active about  $1/4$  of the time. Let  $p$  be used to denote the probability that a given channel is active. In this case, the probability,  $p(n)$  that at any given time,  $n$  channels will be active among  $K$  channels is given by the binomial distribution.

The rms value of the equivalent test tone, when equal volume voice channels are multiplexed, is obtained by first determining the distribution function of  $E/U$  with  $n$ , the number of active channels as a parameter. This distribution is obtained by actually superimposing a number of voice signals at baseband, for it has been found that the frequency separation of the voice channel does not materially affect the distribution of the multiplexed voice signals.  $E$  is the rectified instantaneous voltage of a multiple of baseband voice signals, and  $U$  is the rms voltage of a single voice channel. The distribution of  $E/U$  is approximately normal when  $n$  is 64. From the cumulative distribution of  $E/U$ , it is possible to specify the value of  $E/U$  that is exceeded with a specific probability. The overload level so

determined corresponding to the specified probability, or the overload expectation  $\epsilon$ , is one of the critical design parameters for SSB multiplexed voice signal systems. The overload level in db above the rms voltage of a single voice signal can be easily obtained as a function of the number of active channels,  $n$ , with the overload expectation,  $\epsilon$ , as a parameter.

By considering progressively smaller values of  $\epsilon$  it is possible to obtain a limiting curve of the overload level in db above the single rms speech voltage plotted as a function of the number of active channels,  $n$ . The multi-channel peak factor is obtained by converting the limiting value of the overload level to instantaneous speech voltage level in db above the rms speech voltage of  $n$  channels. This can be readily done by dividing the limiting value of each  $E/U$  corresponding to each  $n$  by  $\sqrt{n}$ . The multi-channel peak factor for  $n$  active channels reaches a constant value of 13.2 db at about  $n = 100$ .

The multi-channel peak factor enables the designer to obtain the largest instantaneous peak voltage excursion of multi-channel voice signals from the rms value. From this peak factor one can obtain the peak instantaneous power of multi-channel voice signals above the average power. The latter can be expressed as  $n$ , number of active channels, times the average power of the single voice channel. Combining the information derived from the multi-channel peak factor and the average power of the voice signals, one can determine the peak instantaneous power of the multi-channel voice signals as a function of the number of active channels,  $n$ . This relation can be expressed in terms of  $K$ , the total number of voice

channels, by specifying  $p(n)$  to be 0.99, for example. The latter relation between instantaneous peak power of multi-channel voice signals plotted as a function of  $K$  will be called the instantaneous load capacity.

The rms power of the test tone is determined by subtracting 3 db from the instantaneous peak load capacity. It has been found convenient in the design of multi-channel voice systems to specify the design parameter in terms of the test tone rather than the instantaneous load capacity.

The case of the equal volume channels has been considered, and a similar relation for the unequal volume multi-channel system is handled at the expense of further analytical sophistication.

## 7.2 Companding and Compression Techniques

Some of the more important characteristics of the single and multi-channel voice signals have been discussed. Methods have been devised to improve the performance of voice communications systems; and in all cases the improvement is obtained by operating on some characteristic of the voice signal prior to transmission and performing the inverse operation at the receiver. Well known are pre-emphasis and voice encoding schemes in which the speech spectrum at the high frequency portion is of importance. As previously discussed, the maximum intensity occurs between 400 and 800 cps, so in the case of pre-emphasis the intensity level of the higher frequencies is increased resulting in an improvement depending upon the degree of pre-emphasis and the signal-to-noise ratio. In the case of digital encoding of voice signals a method has been devised to sample the lower and upper portion of the voice spectrum separately and

then digitize them independently. This results in pulse rates of 20,000 to 40,000 bits per second giving rise to voice quality equivalent to 128 levels sampled 8000 times a second.

The method of voice system improvement of special interest here is companding. Companders operate on the variation of the speech signal intensity and rate of change. Speech intensity varies from syllable to syllable and even within syllables. In terms of energy, there are consonants that have 30 db less energy than the strong vowels, even under constant volume conditions. In a practical system, the volume of the speakers vary. To prevent the transmitter from overloading and to improve the signal-to-noise ratio of the weak speakers, the volume of each voice channel must be adjusted. In the early days of radio telephone, the technical operators used a volume meter to monitor and adjust the speaker volumes for optimum transmission. However, this did not provide a satisfactory means of compensation for the intensity fluctuations within words.

The theory of companders, which consist of a compressor and an expander, is well known. Briefly, the compander crossover level is the average speech power in dbm zero at which the compressor introduces no compression. Thus, if the average speech power exceeds the crossover level it will be attenuated, proportional to the speech power. Yet, if the average speech power is less than the crossover level, the reverse is true.

Forward and backward control methods are used to generate the control information. In the case of the forward method the control voltage is approximately proportional to the envelope of the speech energy and is

obtained from the input to the compressor preceding the circuit that provides the variable loss in the compressor. The control voltage is generated by first passing the speech signal into a nonlinear circuit whose output is some root of the input signal. The output of the nonlinear circuit is the input to a rectifier whose output is then low pass filtered with the appropriate cutoff frequency. In the case of backward control, the variable loss circuit is used to provide the nonlinear rooting operation. In either case, the compression ratio is defined as the ratio of the compressor output increment to the input increment in db.

The expander, located at the receiver, operates on the demodulated signal in exactly the inverse manner to the compressor. It is therefore reasonable to define an expansion ratio which is the inverse of the compression ratio. If a compressor is placed in tandem with an expander, ideally the final output will be identical to the input.

The syllabic compander is designed to have an attack time of 3 to 5 milliseconds and a recovery time of 30 to 50 milliseconds. It is known that vowel sounds seldom build up to their peak intensity in less than 1 or 2 milliseconds, so an attack time of 3 milliseconds will suppress the syllabic peaks. The release time is sufficient to extend over the duration of the syllable. The instantaneous speech voltages that fluctuate more rapidly than the attack time will not be acted upon by the compressor. It is not reasonable to extend the release time much beyond the duration of 30 to 50 milliseconds, for the expander will continue to operate at low loss after the speech burst has ceased. This condition results in the listener hearing the noise after each rise in speech intensity.

The noise improvement of a compander is the signal-to-noise improvement at the output achieved by the instantaneous losses introduced by the expander. The variable loss of the expander is controlled by the average compressed speech power, and varies according to whether the compressed signal represents a pause in speech or not. During a speech pause the compressor can be assumed to be in steady state, so the compressed signal received at the expander consists of channel noise only, which is averaged by the expander control circuitry to determine the loss. The expander time constant is short, and the noise suppression is effective, resulting in noise improvements of the order of 25 db.

For weak consonants the compressor's action which boosts the signal level improves the signal-to-noise ratio in the channel resulting in an enhanced noise improvement. In the case of a vowel with high signal intensity, the speech signal masks the noise and the noise improvement is of the order of a few db's at most.

Syllabic companders have been found to be effective in reducing the multi-channel peak factor and in the reduction of the multi-channel peak load capacity in systems with a small or medium number of channels. The companding of each channel becomes less effective as the number of channels increase in terms of multi-channel peak load reduction. For example, Bell System's N1 carrier system incorporates companders. In the case of the compressor a backward control system is used and the expander is a forward control type. The crossover level is at 5 dbm, and 20 to 28 db noise advantages have been derived.

The companding technique, which is of considerably more interest to digital communication techniques, is the method in which the variable gains of the compressor and expander are changed instantaneously. With instantaneous companding, nonlinear coding of the speech signal is achieved. This coding method is a PCM technique in which nonuniform quantization is provided for small signal amplitudes in comparison to the large amplitudes, resulting in an overall improvement of speech quality in the channel. The nonlinear encoding of speech samples can be conveniently achieved by an instantaneous compander preceding a linear analog-to-digital converter.

The distribution of the talker volume at the zero transmission level, and the instantaneous voltage distribution of speech are taken into consideration in determining the entire range of instantaneous speech voltage samples that require encoding. In terms of speaker volume distribution, which has a standard deviation of approximately 5.8 db, + 13 db about the average volume will include roughly 98% of the speakers.

The upper clipping level of the strong speaker and the low clipping level of the weak speaker determines the range of the instantaneous speech voltage samples. The upper clipping level can be taken 13 db above the mean power of the strong speaker and maybe even less. The low level clipping is determined by specifying the mean square value of the signal to be 23 db above the power of a unit of quantization. This results in 62 db of power spread, and in terms of instantaneous voltage, a spread of 2520. Using uniform quantizing units, 11 bits is required to provide 2048 levels. Using nonlinear transformations, such as logarithmic, hyperbolic or exponential mappings, one can achieve similar signal-to-quantizing noise ratios with only 7 bits.

## Section 8

### STRONG SIGNAL INTERFERENCE

An important problem in multiple access is the ability to accommodate strong and weak stations through a common peak-power limited repeater satellite. It is essential that the satellite power be shared equally among the messages. Thus a large station transmitting ten messages should take no more than ten times the power of a small station which sends only one message. (We assume that the message rates are the same.) The quality of the received signal is, however, dependent on the down link characteristics of each receiving station. Hence, a station which has a large antenna, or one which has a better receiver noise figure, will be able to receive signals of superior quality than a station which has inferior receiving characteristics.

There are a number of ways which can be used to multiplex strong and weak stations so that each receives a fair share of power, all of which require some control of the signal in the up-link.

Among the common techniques are ground station power control, time division multiplexing using net synchronization

(TDM), and pulsed pseudo-noise signal transmission with time and frequency hopping.

Ground station power control may be an easy way of maintaining equal power at the satellite. Its effectiveness depends upon:

- (1). Accuracy of control of transmitter output power;
- (2). Antenna tracking accuracy;
- (3). Range accuracy.

The transmitter output power can be controlled by a simple feedback loop. Such loops are not difficult to build for slow varying power fluctuations (i. e. . less than 3 to 5 cps.) A wide bandwidth loop might present design difficulties. Using such a feedback loop, one should be able to control the power to within approximately 1 db.

The power fluxuations due to variations in antenna gain with pointing error will depend on the beamwidth and tracking accuracy of the antenna. A power fluctuation of approximately 1 db is a reasonable estimate for antennas of the type used for synchronous satellite operation.

Given sufficient time, range accuracy to a synchronous satellite can be determined to within a few feet. For this reason, power fluctuations due to range uncertainty will be assumed negligible in comparison to the previously discussed errors.

Therefore, using ground station power control, a variation of 2 db seems easily obtainable. With additional complexity in the feedback loop and the antenna tracking system, power variations of considerably less than 2 db are possible.

Time Division Multiplexing (TDM) solves the power control problem by essentially eliminating it. Here, each station is assigned a time slot in which it transmits with no interference from other stations.<sup>(21)</sup> The problem here is ultimately one of establishing accurate timing. This could be accomplished by a satellite time standard that transmits one or more timing bursts during each frame. Since only relative time is needed, the time accuracy is determined by the range uncertainties with respect to the various stations as well as variations in the velocity of propagation due to atmospheric effects. After sufficient measuring, range uncertainties of only a few meters can be obtained. A 3 meter rms range error means of course an rms time error of .01 sec.

Making a standard correction for propagation effects will result in an rms timing error of approximately 0.03 sec. at 2 gc.<sup>(19)</sup> This number can be reduced by the use of more sophisticated correction techniques.

For the purpose of this discussion we will assume that a hard limiter is used in the satellite. Pulsed pseudo-noise transmission gives the weak station a way of avoiding the suppression effects of the strong one.

As an example, assume that  $(K-1)$  strong stations and one weak station each of duty factor  $d$  are transmitting signals. Then  $(1-d)^{K-1}$  is the probability that the  $(K-1)$  strong stations will be off during the interval that the small station uses the satellite. During this interval the transmitted signal in the down link is  $P_S$  and the thermal noise power is  $2WN_0$ . The probability that a very strong station captures the limiter when the small signal is present is  $(1-(1-d)^{K-1})$ . Here, clutter is generated of power  $P_S$  and thermal noise is added at the receiver of power  $2WN_0$ ; the received signal power component is assumed to be zero. From these results a lower bound on the performance in the presence of strong signals can be obtained.

From an error probability point of view, this type of calculation would indicate that performance is necessarily poor unless the duty factor is chosen to be very small. Such a duty factor, however, would lead to extremely inefficient utilization of bandwidth and on-board power. The probability of error by itself is, however, a poor indication of voice intelligibility particularly when burst errors are concerned. Interesting results on the effects of periodic burst interference on voice recognition are discussed in the next paragraph.

In choosing a modulation technique for a voice communication system to operate in an asynchronous multiple access environment, the results of articulation tests performed under certain interfering conditions are of interest. The experiments were conducted under varying probabilities that an actual interference would occur. Assuming the interference to be periodic, articulation tests were made for various interference rates. The results show that for a specified probability of interference and voice signal-to-noise ratio, there is an interference rate that maximizes the articulation test scores.

When the noise time function, or the probability of interference, is 0.2 for various voice signal-to-noise ratios during the interference, the articulation test scores of Miller and Licklider are:

Interference Rate/S/N	-18db	-9db	0db	+9db
1000 cps	15	57	82	92
100 cps	55	82	92	94
10 cps	96	98	99	98
1 cps	86	92	96	97

For a probability of interference of 0.5, the articulation test scores decline as shown below:

Interference Rate/S/N	-18db	-9db	0db	+9db
1000 cps	0	24	63	83
100 cps	0	39	75	86
10 cps	71	82	93	95
1 cps	53	70	84	92

The articulation test score method is important in designing communication systems. However, this may not reflect speaker recognition and other qualities essential to a good voice channel.

## Section 9

### RECOMMENDATIONS

In consonance with the conclusions reached in Section 6, the important recommendation to be made is that an experimental program over a SYNCOM channel be initiated as a follow-on to this contract. A minimum of additional theoretical and systems analysis will be performed in support of this proposed program.

#### 9.1 Experimentation

IBM feels that an experimental program is an integral part of any extension of the work done under this contract. Although this study has demonstrated the theoretical feasibility of PN multiplexing as applied to a random-access SYNCOM satellite system, the practical feasibility has not been demonstrated. Considerable experimental work must be performed before our knowledge of PN multiplexing techniques is comparable to our understanding of conventional modulation techniques.

IBM feels that this deficiency can be rectified by a program employing, primarily, satellite experimentation combined with some laboratory work and computer simulation.

Some experiments using CW, pseudo-noise transmission over a repeater satellite have been reported,\* although results have not been reported. IBM recommends that experiments over SYNCOM be initiated which use Pulsed

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\*IEEE International Meeting, March 1964, N. Y., N. Y.

Pseudo-Noise Multiplexing and Matched Filter Reception. The results of this study show that these techniques are attractive both from on-board power requirements as well as from operational and implementation considerations. These experiments will also demonstrate the usefulness of wide deviation pulse rate modulation (or digital pulse-position modulation) in multiple access satellite communications. This form of modulation has received little attention in satellite applications because, in its conventional form, it requires low duty factor pulsed signals which are not compatible with a peak-power limited repeater. However, this type of modulation, when incorporated with pulsed PN multiplexing and matched filter reception, loses its deficiency and becomes extremely attractive as an efficient way of exchanging bandwidth for on-board power.

Most studies on optimum modulation techniques have neglected callup logic and callup procedures. We have included callup logic in this study and have shown a relatively simple PN callup technique using a maximal length sequence generator and a matched filter receiver. This technique is applicable to conventional and PN multiplexing and hence is of general interest. The satellite experiments should therefore include this recommended callup procedure and test its usefulness. In particular, it is of interest to slow the PN bit rate during callup relative to that used during message transmission in order to reduce the chances of false calling. This mode of operation will also demonstrate the feasibility of accommodating data and voice on a common channel. The reduced data rate will reduce the error probability as required, when data rather than voice is the message.

The matched filter receiver permits asynchronous reception of the voice message. In many applications this is a desirable mode of operation, particularly where small mobile ground stations are of interest. Experiments should therefore demonstrate this type of operation.

The matched filter receiving system is compatible with frequency hopping. If bandwidth is available this mode of operation reduces clutter and in addition permits the multiplexing of strong and weak stations. The use of pulsed pseudo noise signals also helps to reduce the degradation caused by a strong signal in the up-link. Measurements of this degradation in the voice signal should therefore be made with the presence of a hard limiter and also with AGC. Parts of this measurement can be made over a SYNCOM link and parts can be made in the laboratory. It is expected that for voice, subjective tests will be most useful.

In all of these experiments, it is recommended that the interfering multiplexing signals which will be generated and transmitted should consist of the PN signals that would ultimately be used. This can be accomplished simply and cheaply.

It is of value to multiplex PN signals with conventional narrowband signals at the satellite repeater. This measurement will show the interference effect of PN transmissions on conventional modulation systems as well as conventional modulation on the PN systems. These measurements should serve to alleviate any unfounded fears that PN modulation will disrupt conventional communications.

In summary, IBM recommends that NASA pursue a satellite experimental program using pulsed PN transmission and matched filter reception. This program will give answers to the following questions.

- The degree of agreement between the theory of PN multiplexing (developed during this program) and experimental results.
- The efficiency with which pulsed PN transmissions and matched filter reception of pulse rate modulated signals use a satellite repeater (equal power multiplexing is assumed).
- The distortion introduced in a voice channel by strong signal interference when pulsed PN multiplexing is used combined with rapid frequency hopping (i.e., burst noise distortion on voice signals).
- The effect of a hard limited and AGC on pulsed PN multiplexing using frequency hopping.
- The effect of slowing the PN signaling for data transmission and multiplexing with high-speed PN voice modulated signals.
- Reliability of PN callup procedure.
- Degree of interference of PN signals with conventional systems and vice versa.
- The effect of AM to PM conversion, doppler, etc., on pulsed PN signals using matched filter reception.

Attempts should be made to utilize any existing experimental data derived from SYNCOM experiments. PN modulation is compatible with existing ground stations and hence serious interface problems are not anticipated.

## 9.2 Systems Analysis

The results of the study are generally applicable to a variety of communications systems, for the modulation comparisons were performed with few restrictions on the communications system parameters and requirements. The system analysis and design required in specifying the interface equipment between the SYNCOM system and the matched filter equipment will use the basic relationships generated in this study.

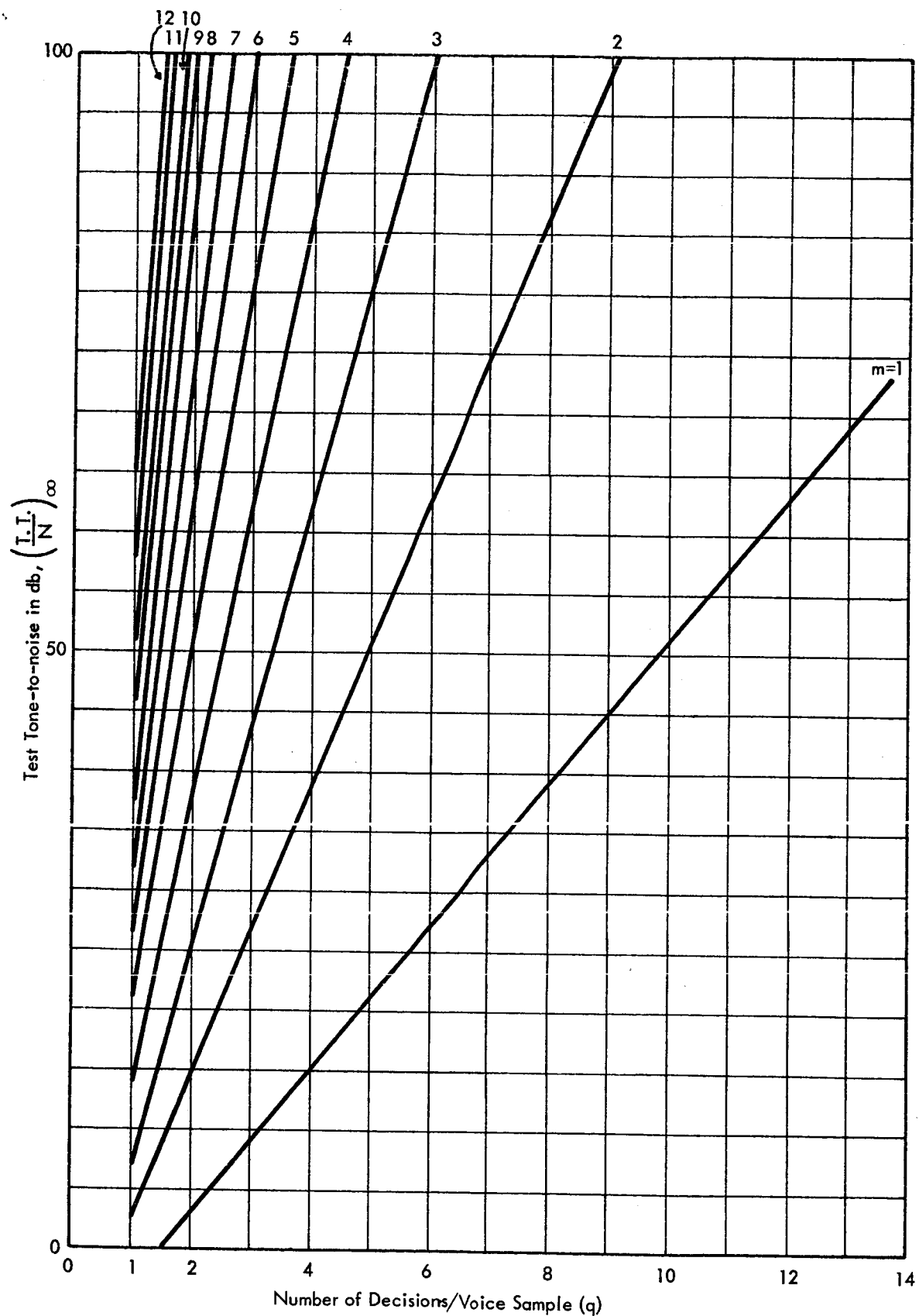


FIGURE 1. TEST TONE-TO-NOISE VS NUMBER OF DECISIONS PER VOICE SAMPLE  $(T.T./N)_t = 3(2mq - 3)$

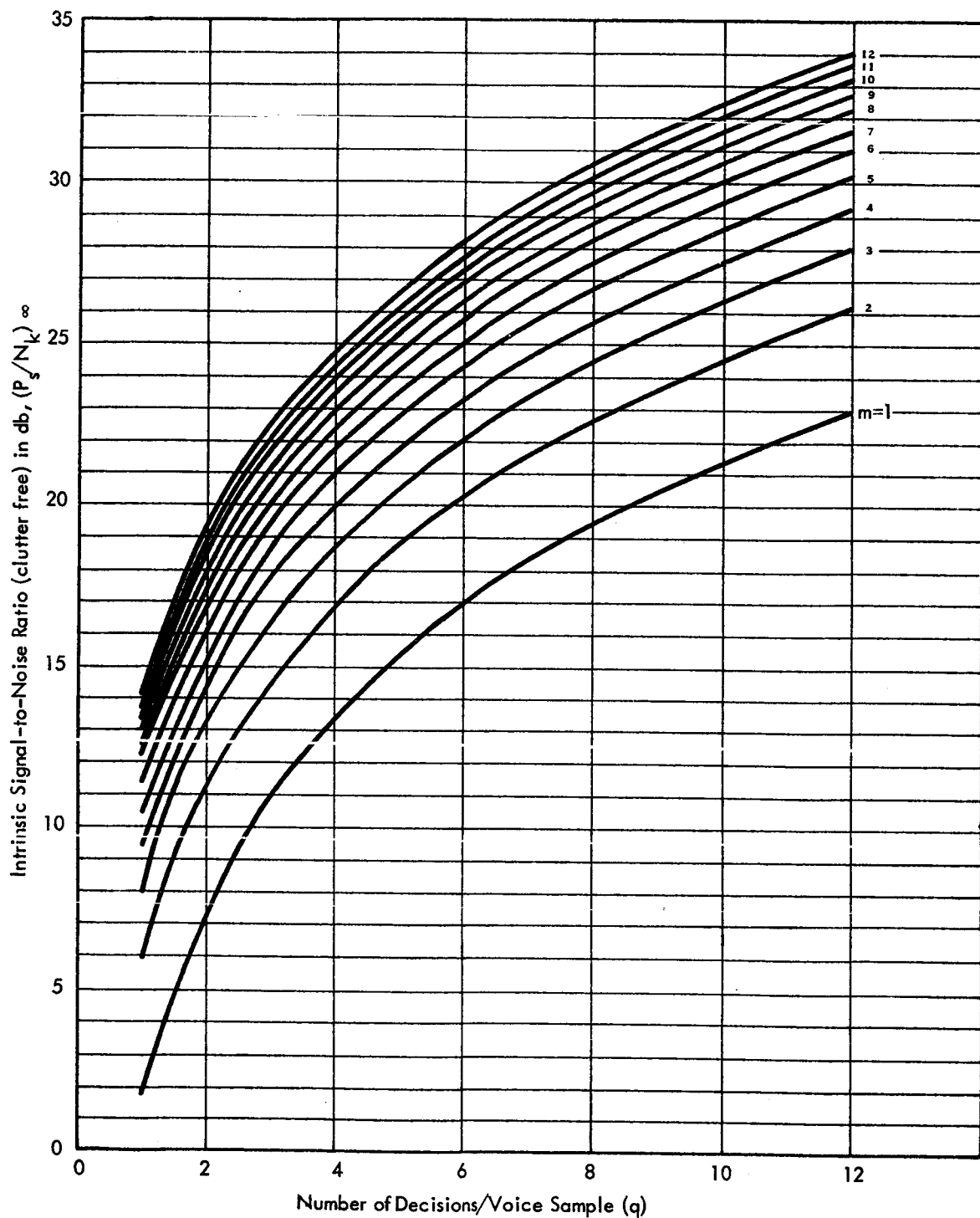


FIGURE 2. INTRINSIC SIGNAL-TO-NOISE RATIO vs. NUMBER OF DECISIONS PER VOICE SAMPLE AT KNEE FOR  $2W \gg P_S/N_o$

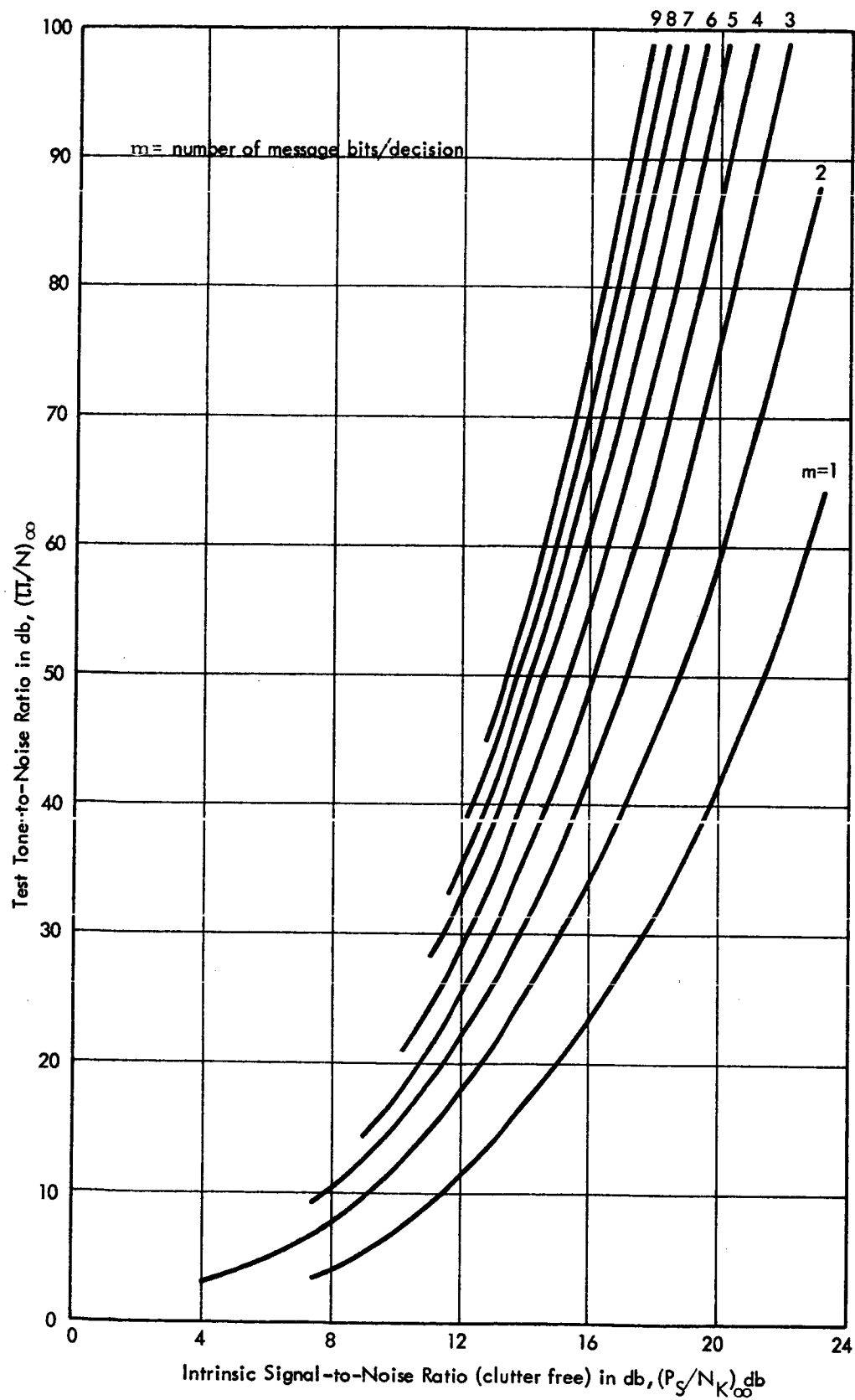


FIGURE 3. TEST TONE-TO-NOISE RATIO VS INTRINSIC SIGNAL-TO-NOISE RATIO

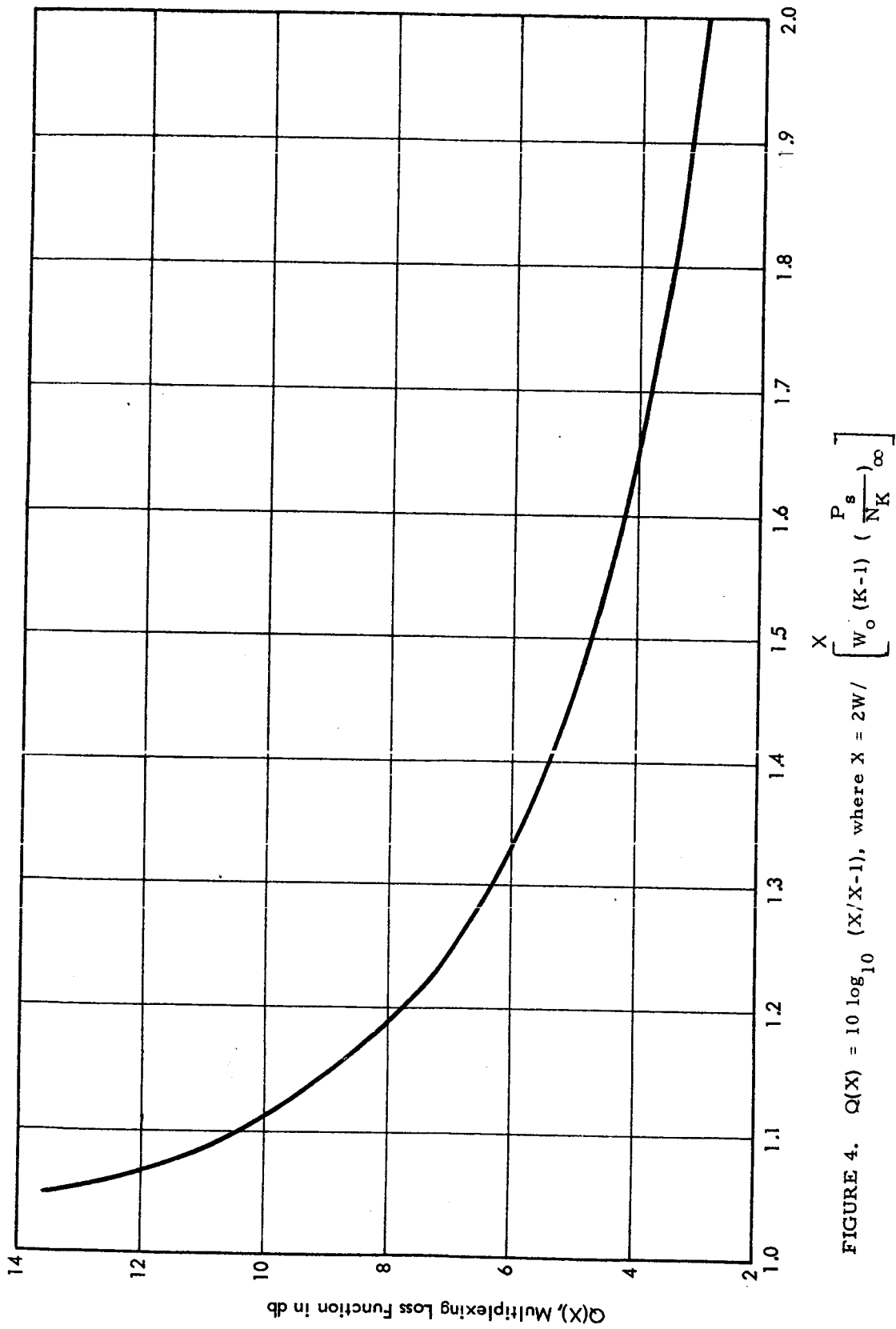


FIGURE 4.  $Q(X) = 10 \log_{10} (X/X-1)$ , where  $X = 2W / \left[ W_0 (K-1) \left( \frac{P_s}{N_K} \right)_\infty \right]$

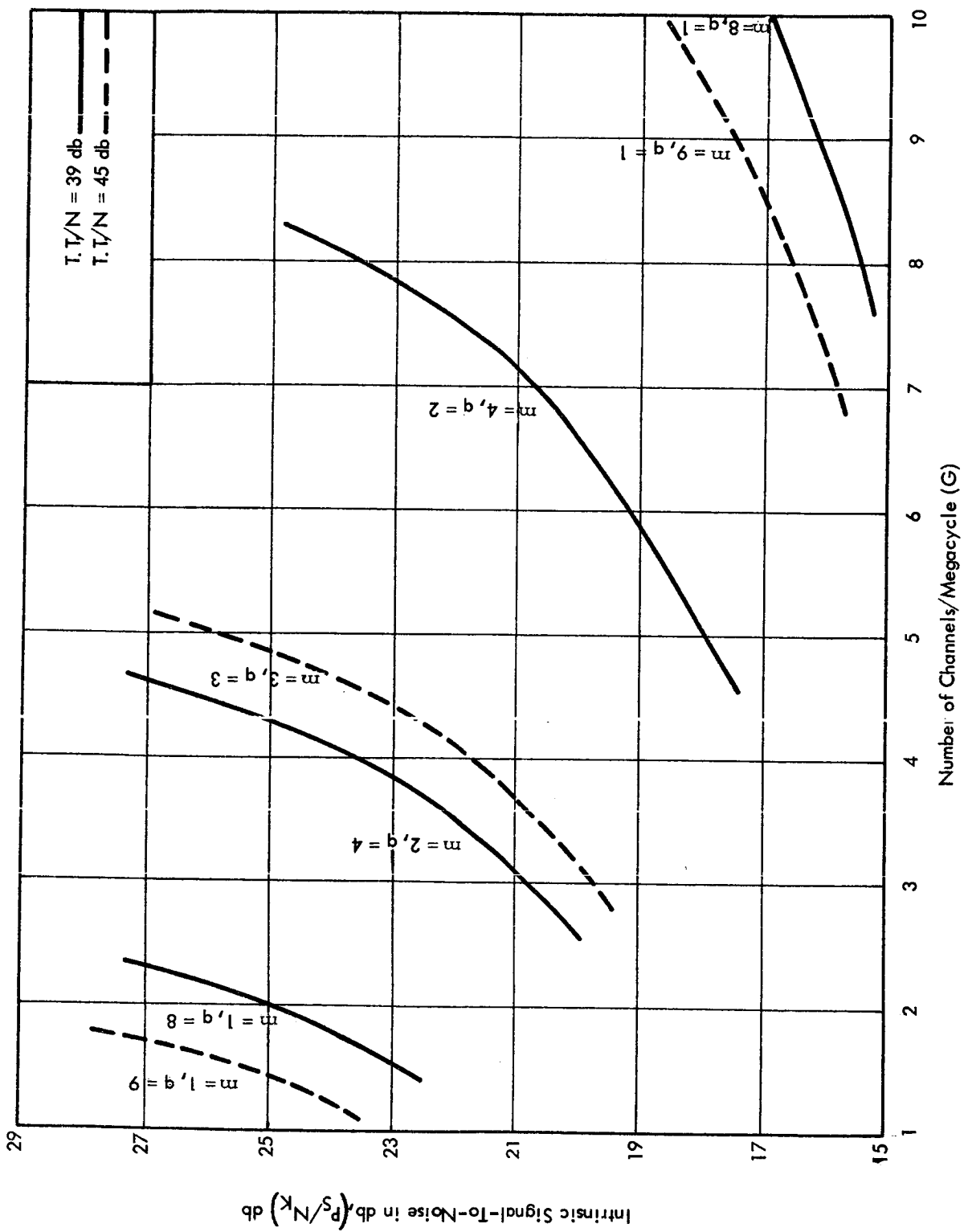


FIGURE 5. INTRINSIC SIGNAL-TO-NOISE RATIO VS CHANNELS PER MEGACYCLE

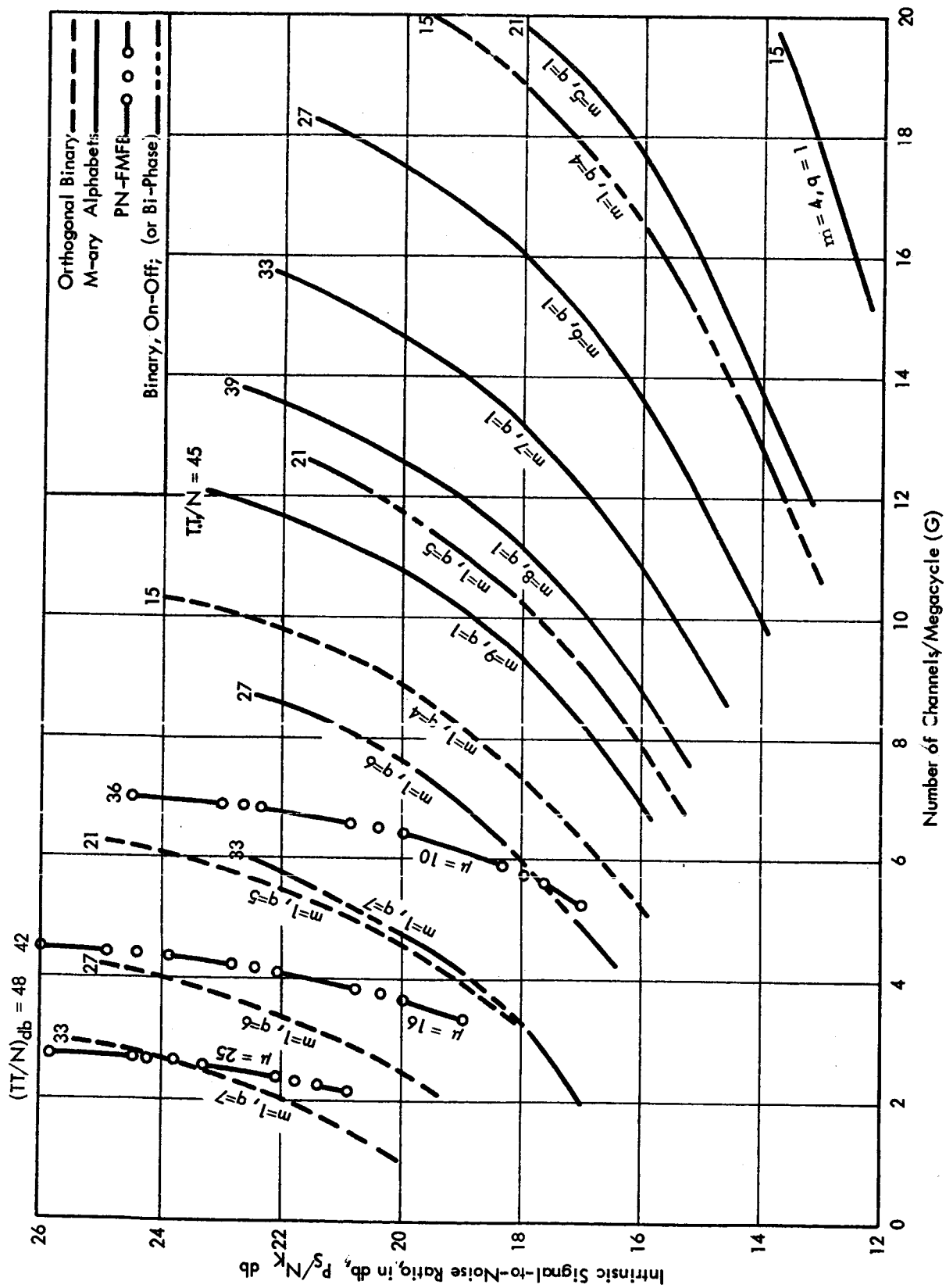


FIGURE 6. INTRINSIC SIGNAL-TO-NOISE RATIO VS CHANNELS PER MEGACYCLE

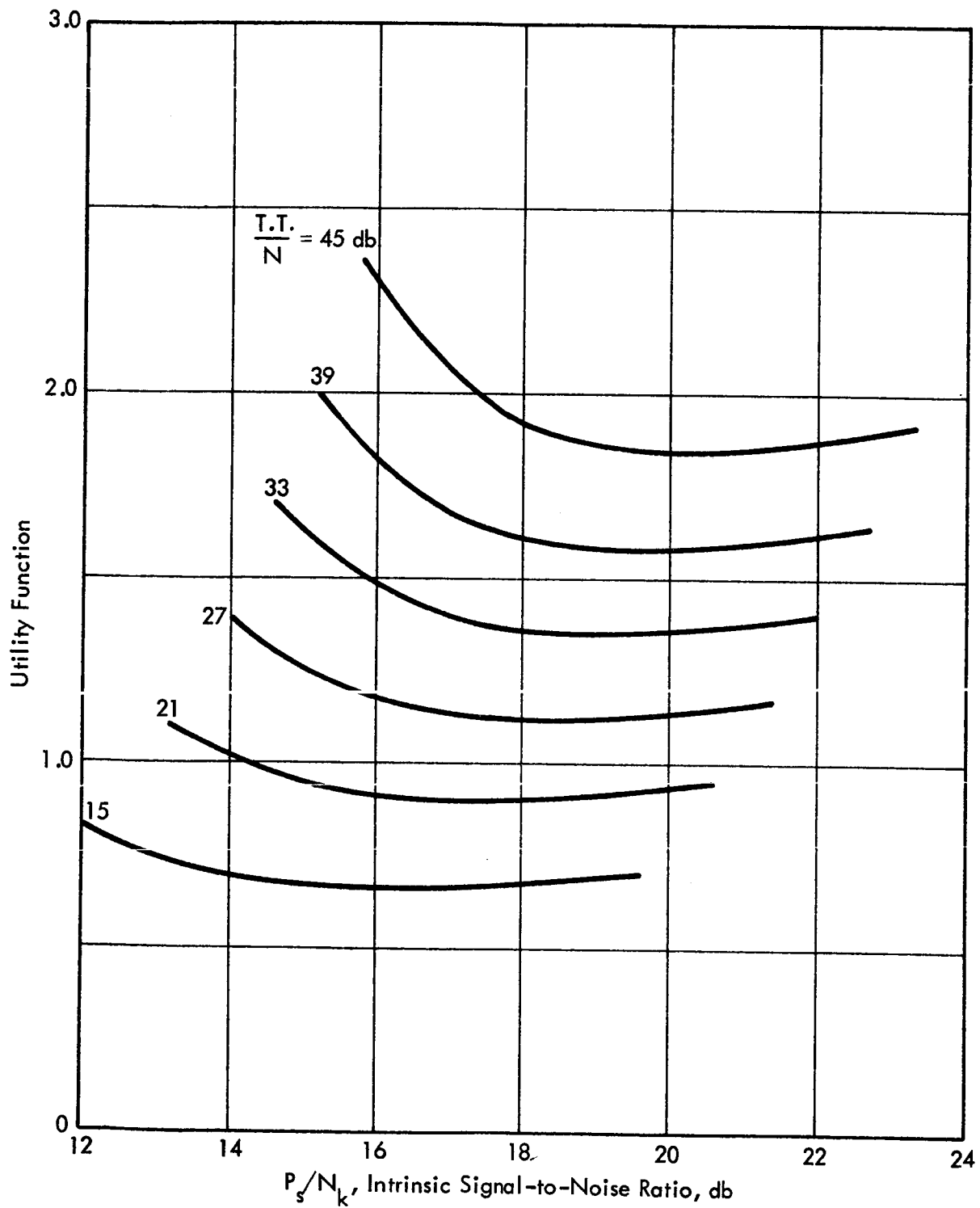


FIGURE 7. UTILITY FUNCTION VS INTRINSIC SIGNAL-TO-NOISE RATIO FOR PN WITH M-ARY ALPHABET

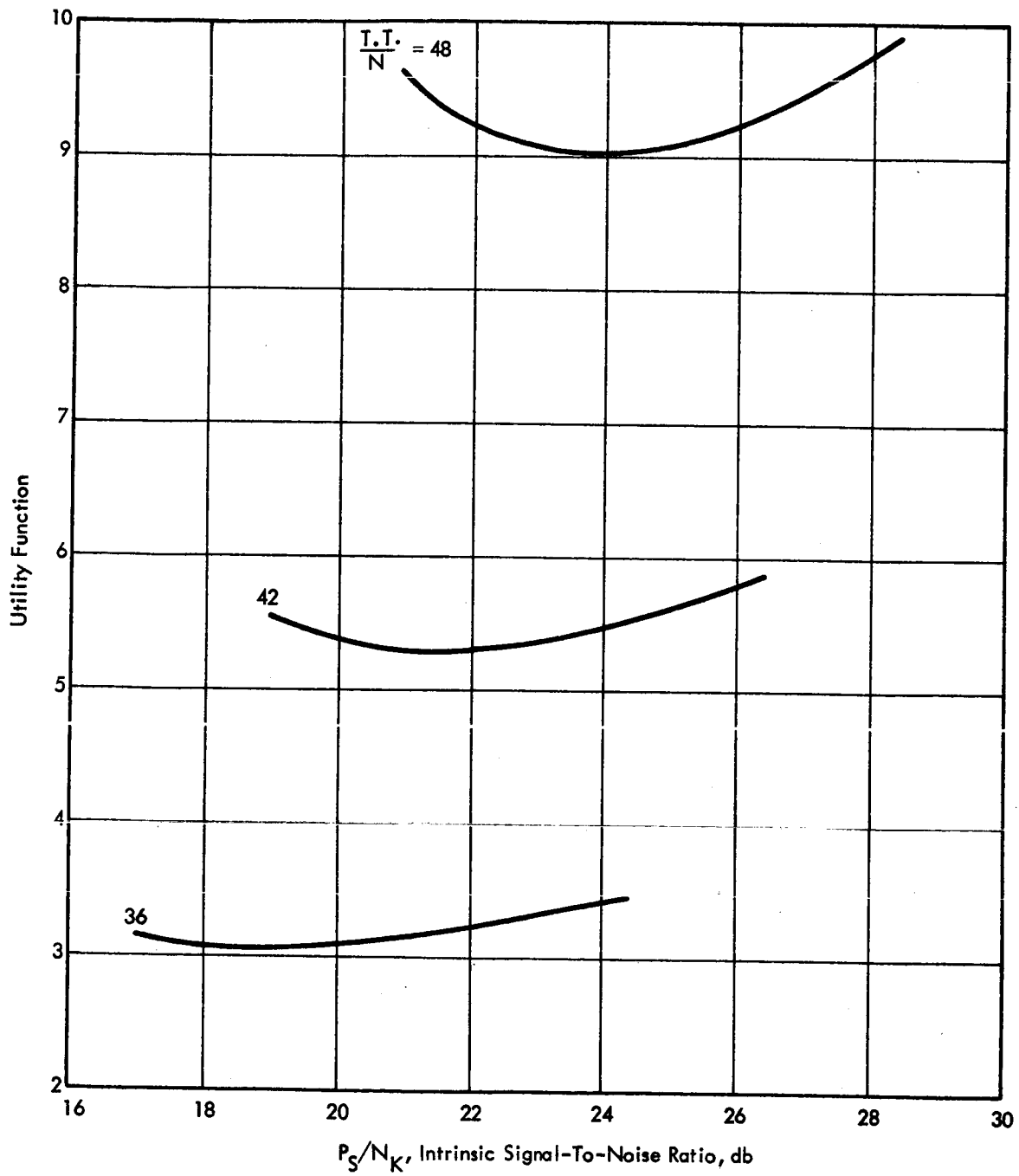


FIGURE 8. UTILITY FUNCTION VS INTRINSIC SIGNAL-TO-NOISE RATIO FOR PN-FMFB

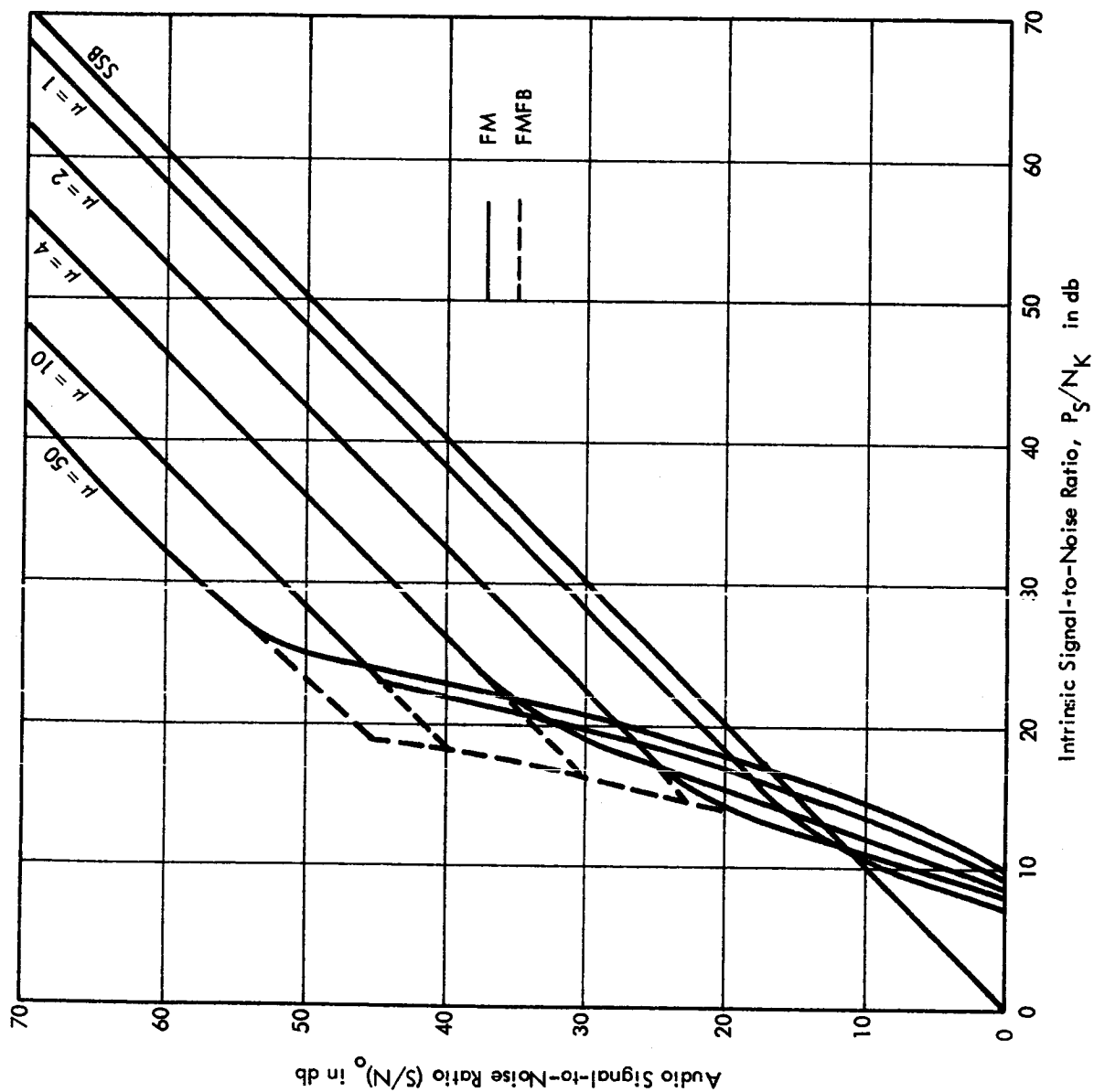


FIGURE 9. AUDIO SIGNAL-TO-NOISE RATIO VS INTRINSIC SIGNAL-TO-NOISE RATIO (CLUTTER FREE) - FM AND FMFB

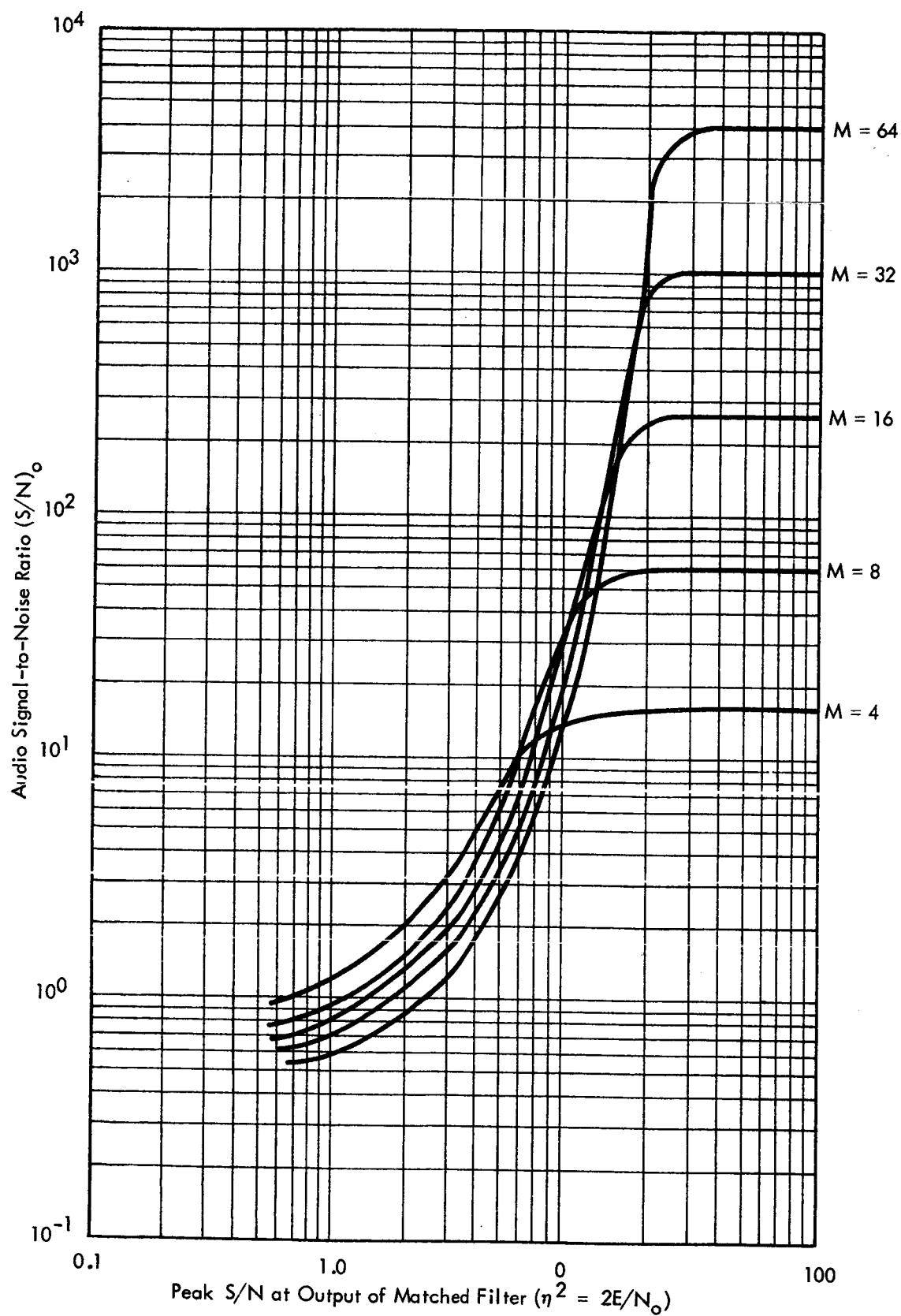


FIGURE 10. AUDIO SIGNAL-TO-NOISE RATIO VS MATCHED FILTER OUTPUT SIGNAL-TO-NOISE RATIO

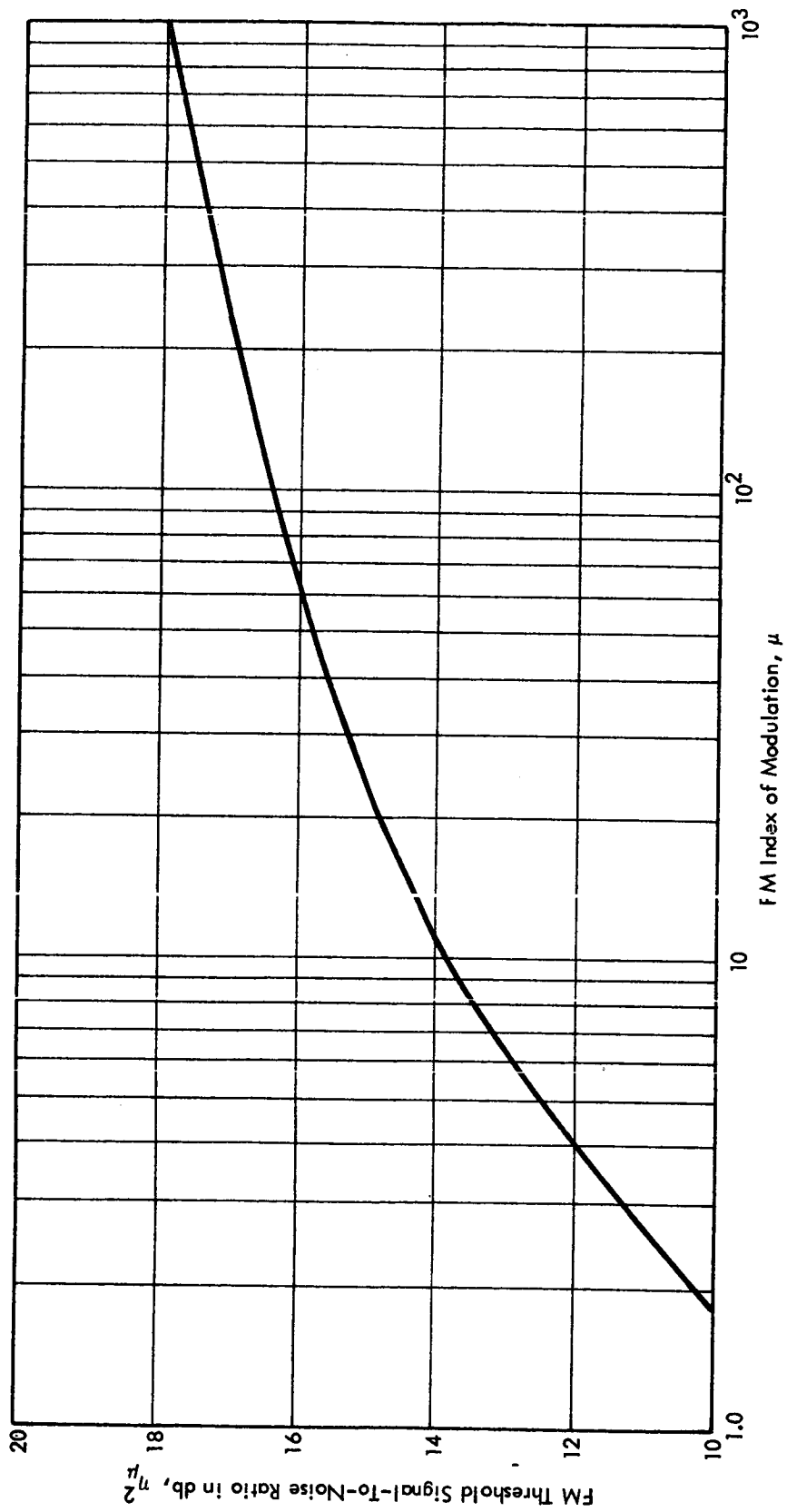
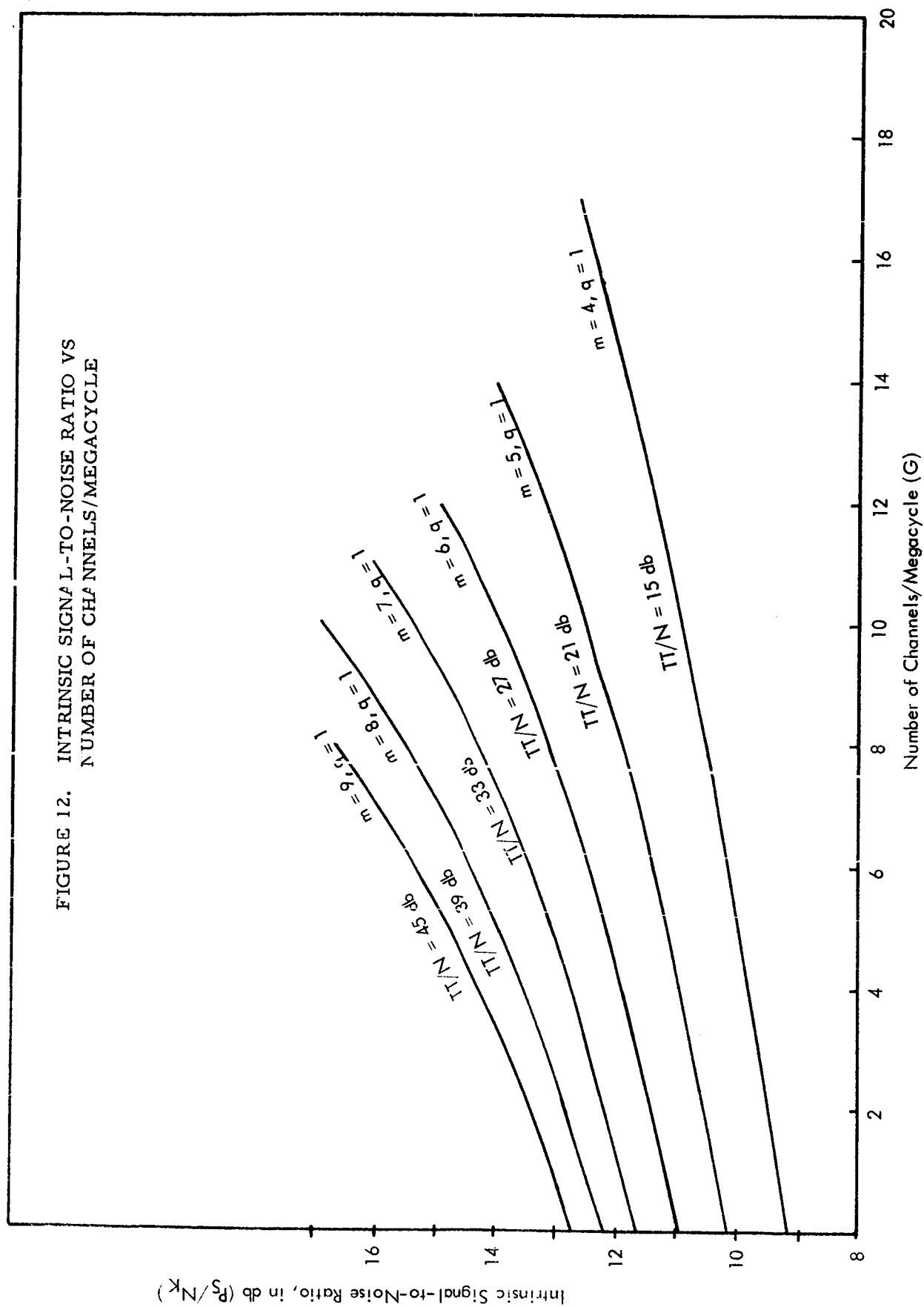


FIGURE 11. FM THRESHOLD SIGNAL-TO-NOISE RATIO VS FM INDEX OF MODULATION

FIGURE 12. INTRINSIC SIGNAL-TO-NOISE RATIO VS  
NUMBER OF CHANNELS/MEGACYCLE



## Appendix A

### ANALYSIS OF PN-MULTIPLEXING SYSTEMS USING DIGITAL SIGNALING TECHNIQUES

It is the purpose of this appendix to develop the necessary relationships to allow PN-multiplexing systems using quantized sample data (i. e., digital) modulation techniques to be compared to a reference system. The reference system will be the single sideband-frequency division multiplexed system (SSB-FDM). Three reasons for the selection of the SSB-FDM system as a reference are (1) It makes the most efficient use of channel bandwidth and therefore maximizes the number of talkers per megacycle; (2) It has been used in the past for the comparison of conventional modulation techniques; (3) It is the best understood voice communications system in existence.

Sections A. 1 and A. 2 review the concepts of audio signal-to-noise ratio and the test tone-to-noise ratio (T. T. /N) respectively as they relate to this problem with adequate references to the literature for more detailed discussions. Relationships are obtained for the audio signal-to-noise ratio, and (T. T. /N) resulting in an expression for (T. T. /N) as a function of the peak square signal to mean square noise ( $\gamma_p^2$ ) at the output of the detector. Section A. 3 makes the final step and relates (T. T. /N) to the SSB-FDM reference system. The final section, A. 4, derives the final expressions used for computing the graphical data necessary for the modulation techniques comparisons.

### A. 1 Audio Signal-to-Noise Ratio

In digital voice communications systems, in which the voice samples are pulse code modulated, there are two causes of audio distortion: decision error noise and quantization noise. An important measure of system performance is the ratio of mean square signal to mean square error. (This is the fidelity criterion used for analog transmission.) If  $S_n$  is the output at the  $n^{\text{th}}$  sample time in the absence of noise and  $O_n$  is the actual  $n^{\text{th}}$  output sample, the error in the  $n^{\text{th}}$  sample will be  $\epsilon_n = S_n - O_n$ . The mean square signal to mean square error ratio will then become,

$$\frac{S^2}{\epsilon^2} = \frac{\lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N S_n^2}{N}}{\lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N \epsilon_n^2}{N}} \quad (\text{A-1})$$

This measure of audio quality will be defined as the audio signal-to-noise ratio,  $S/N$ .

In digital voice transmission systems such as described here, the audio signal is sampled and quantized into one of  $M$  values. The  $M$  values are then transmitted as one of  $M$  possible

waveforms. At the receiver, the waveforms are detected and the audio signal is retrieved by appropriate low pass filtering.

The audio signal-to-noise ratio will depend upon the number of quantization levels,  $M$ , and the probability of incorrect detection,  $\alpha$ . This probability will in turn depend upon the signal-to-total-noise ratio at the output of the correlation receiver and on the method of detection. (The noise here consists of thermal noise plus clutter.)

First, an expression for  $S/N$  as a function of  $M$  and  $\alpha$  will be given. Although the method of detection may drastically change the value of  $\alpha$ , for all practical purposes the relationship for audio signal-to-noise remains unchanged.

A mathematically convenient model for the statistics of the analog source is that the amplitude probability density is flat. This is not restrictive and quite general since an arbitrarily distributed random variable can always be mapped (by means of a nonlinear amplitude transformation which is the inverse of the cumulative distribution) into one which has a flat density function. Furthermore, the original random variable can be recovered by

the inverse transformation. Even more important, the fidelity criterion which will be used is quite insensitive to the probability distribution of the random variable <sup>(13)</sup>.

In Reference 13 it is shown that the audio output signal-to-noise ratio has the functional form,

$$\left(\frac{S}{N}\right) = \frac{M^2 - 1}{1 + 4\alpha_B(M^2 - 1)} \quad (A-2)$$

where  $M$  is the number of levels which partition the range  $(-A, A)$  and  $\alpha_B$  is the bit error rate. It is now essential to calculate the bit error probability  $\alpha_B$  for various methods of transmission. In the  $M$ -ary case it is shown <sup>(13)</sup> that,

$$\alpha_B = \frac{M}{2(M-1)} \alpha \quad (A-3)$$

where  $\alpha$  is the  $M$ -ary decision probability of error. From equations (A-2) and (A-3),

$$\begin{aligned} \left(\frac{S}{N}\right) &= \frac{(M^2 - 1)}{1 + 2M(M+1)\alpha} \\ &\approx \frac{1}{2\alpha + M^{-2}} \quad \text{for } M \gg 1 \end{aligned} \quad (A-4)$$

When bit-by-bit decision is used (i. e. , as in conventional PCM) then for high quality voice we have from equation (A-2)

$$\left(\frac{S}{N}\right) = \frac{1}{4\alpha + M^{-2}} \quad ; \quad M \gg 1 \quad (A-5)$$

where  $\alpha$  is the bit decision error.

In this report, we also consider multiple word decisions, which will also result in some small modification of the expression for  $\frac{S}{N}$ . For large values of  $M$ , i. e. , for good voice quality, the factors which multiply the error probability  $\alpha$  will influence the fidelity expression  $\left(\frac{S}{N}\right)$  only slightly. Thus for the purpose of this study we use equation (A-4) for all the decision procedures studied with the appropriate value of  $\alpha$  recognizing that for  $M \gg 1$  the discrepancy between the exact expression and the one used is of little practical significance.

From equation (A-4) it can be seen that the audio signal-to-noise ratio depends upon decision errors (i. e. , the  $2\alpha$  term) and quantization errors (i. e. , the  $1/M^2$  term) as mentioned in the introduction to this appendix.

Before introducing the concept of (T. T. /N) it will be useful to express S/N as given in equation (A-4) as a function of  $\eta^2$

the correlation receiver average signal-to-noise ratio output.

The probability of error  $\alpha$  is related to the signal-to-noise at the output of a matched filter with greatest-of decision by the equation,

$$1 - \alpha = \int_0^{\infty} y \exp \left\{ -\frac{1}{2} (y^2 + \eta^2) \right\} I_0(\eta y) \left[ 1 - \exp \left\{ -\frac{1}{2} y^2 \right\} \right]^{M-1} dy \quad (A-6)$$

This equation assumes an orthogonal signal alphabet and envelope detection. Under the assumption of large signal-to-noise ratio, equation (A-6) becomes (11), (18),

$$\alpha = \frac{M-1}{2} \exp \left\{ -\frac{1}{2} \eta^2 \right\} \quad (A-7)$$

(This equation is a very good upper bound on the phase coherent case as well, when  $M \gg 1$ .) Substitution of equation (A-7) into equation (A-4) and expressing the results in decibels yields,

$$\left( \frac{S}{N} \right)_{db} = 2.2 \eta^2 - 10 \log_{10}(M-1) - 10 \log_{10} \left\{ 1 + \left[ Q(M)/(M-1) \right] \exp \left( \frac{1}{2} \eta^2 \right) \right\} \quad (A-8)$$

where  $Q(M)$  represents the quantization noise-to-signal ratio,

$$Q(M) = \exp_2(-m_b) \equiv 2^{-m_b} \quad (A-9)$$

$m_b$  = total number of bits in the sample waveform

It is now necessary to extend the results to the case where a decision is made on subparts of the sample waveform. One can define a quantity,  $q$ , equal to the number of decisions per sample waveform. If  $m$  bits are included in each decision, then the total number of bits will be,

$$m_b = mq \quad (A-10)$$

For a sampling period  $T_s = 1/(2W_o)$  and a sub-waveform time duration  $T$ ,

$$T = T_s/q = \frac{1}{2W_o q} \quad (A-11)$$

Therefore,

$$q = \frac{1}{2W_o T} \quad (A-12)$$

Both  $Q(M)$  and  $\eta^2$  are functions of  $q$ . For the quantization noise, it can be easily shown that,

$$Q(M) = 2^{-2m_b} = \left[ 2^{-m} \right]^{2q} = M^{-2q} \quad (A-13)$$

For the case of word decisions,  $q = 1$  and equation (A-13) reduces to the usual expression for  $Q(M) = \frac{1}{M^2}$ . The relationship of  $\eta^2$  to  $q$  will be discussed in Section A. 3.

The next section will briefly discuss the concepts of test tone-to-noise and how it relates to voice communications comparisons.

## A.2 Test Tone-to-Noise Ratio

In order to compare the various modulation methods for a multiple access system, it is convenient to express the audio signal-to-noise ratio in terms of the test tone-to-noise ratio. The channel test tone is a 1 kc sinusoidal tone which has an average power of 1 mw at the toll switchboard, or the 0 dbm 0\* point. The test tone is conveniently used as a voice channel reference signal.

The average power of the signal in the audio channel can be expressed in terms of the test tone as

$$S_o = (T.T.) \cdot X,$$

where T.T. denotes the test tone defined above at 0 dbm 0.

The value of  $X^{(6)}$  at the 0 dbm 0 point is determined by the voice signal characteristics and the required voice quality. It has been found that in an audio channel, which is one of many voice channels multiplexed in a single sideband manner, a 1%

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\* 0 db with respect to a milliwatt at the zero relative level point

of the time overload does not impair the voice quality. For the unequal speaker volume case using this criteria of 1% permissible overload, the instantaneous load capacity of a single voice channel in terms of the rms power of a test tone should be 9.5 dbm at the zero relative point. This determines the value of X in db to be 9.5 db.

Hence,

$$\left( \frac{T. T.}{N} \right)_{db} = \left( \frac{S}{N} \right)_{db} - 9.5 \quad (A-14)$$

In the actual telephone voice channel it is reasonable to assume  
(15)

that a 3.1 kc bandwidth is used and that psophometric weighting of the noise spectrum provides a  $\left( \frac{T.T.}{N} \right)_{db}$  improvement of 3.5 db. Hence we have

$$\left( \frac{T. T.}{N} \right)_{db} = \left( \frac{S}{N} \right)_{db} - 6.0 \quad (A-15)$$

Therefore, from equation (A-8)

$$\left( \frac{T. T.}{N} \right)_{db} = 2.2 \eta^2 - 10 \log_{10} (M-1) - 10 \log_{10} \left\{ 1 + \left[ Q(M)/(M-1) \right] \exp (\eta^2/2) \right\} - 6 \quad (A-16)$$

### A. 3 $\left(\frac{T.T.}{N}\right)$ Referred to An SSB-FDM, K Channel System

To derive an expression for the  $\left(\frac{T.T.}{N}\right)$  in terms of the reference K channel SSB-FDM system, one must first derive an expression for the peak signal power-to-mean square noise ratio at the output of the matched filter,  $\gamma_p^2$ , in terms of the signal parameters at the input to the matched filter receiver.

(This is performed in detail in the Addendum at the end of the appendixes section.) From this, one can derive an expression for  $\gamma_p^2$  as a function of the equivalent signal-to-noise ratio  $\frac{P_S}{N_K}$  in a K channel SSB-FDM system, thereby allowing one to relate  $\left(\frac{T.T.}{N}\right)$  to the reference system.

A brief summary of the clutter calculation in the Addendum is presented here for completeness.

The  $n^{\text{th}}$  signal waveform can be expressed as

$$Z_n(t) = A_n \exp \left\{ j[\omega_0 t + \phi_n(t)] \right\} \quad 0 \leq t \leq T$$

$$n = 1, 2, \dots, M$$

(A-17)

(  $Z(t)$  is the analytic signal representation)

$T$  = duration of waveform (i. e. , the integration time)

$A_n$  = waveform amplitude

$\omega_0$  = carrier frequency

$\phi_n(t)$  = pseudo-random phase modulation

The received signal will also include clutter signals to which the intended receiver is not matched as well as thermal noise.

Hence,

$$Z_R(t) = Z_n(t) + Z_c(t) + n(t) , \quad (A-18)$$

where,

$Z_c(t)$  = clutter signal

$n(t)$  = complex white gaussian process of spectral density

$N_0$  watts/cps.

The clutter signal will be of the form

$$Z_c(t) = \sum_{\substack{p \\ p \neq n}} A_p \exp \left\{ j \left[ \omega_0(t - \gamma_p) + \phi_p(t - \gamma_p) \right] \right\} , \quad 0 \leq t \leq T \quad (A-19)$$

$\gamma_p$  = relative time shift of the  $p^{\text{th}}$  clutter signal with respect to the desired one.

To derive the desired expression for the correlator output signal-to-noise ratio, it will be necessary to assume that the clutter signals have a white gaussian distribution. (This assumption is not required in the Addendum.) Then one can define the total noise power density as

$$N_{ot} = N_{oc} + N_o$$

where

$N_{oc}$  = the noise power density of the clutter signals (watts/cps)

$N_o$  = the thermal noise power density (watts/cps)

One can now use the well known expression for the output signal-to-noise ratio of a matched filter at the instant of match for white gaussian noise (14).

$$\eta_p^2 = \frac{2E}{N_{ot}} \quad (A-20)$$

Assuming the RF bandwidth to be  $2W$ ,

$$\eta_p^2 = \frac{4EW}{N} \quad (A-21)$$

where  $N_{ot} = \frac{N}{2W}$

Assume that  $K$  signals are being received by the matched filter each of which has the same power  $P_k$ . Then the clutter power will be  $(K-1)P_k$  and  $E = P_k T$ , is the energy per waveform. Therefore

$$\eta_p^2 = \frac{4WTP_K}{2N_o W + (K-1)P_K} \quad (A-22a)$$

where  $2N_o W$  is the thermal noise power and  $(K-1)P_k$  is the total clutter noise power. Dividing numerator and denominator by  $2W P_k K$ , we have,

$$\eta_p^2 = \frac{\frac{2T}{K}}{\frac{N_o}{KP_K} + (1 - \frac{1}{K}) \frac{1}{2W}} \quad (A-22b)$$

Since the total power received is  $P = KP_k$  where  $P$  is the satellite power referred to the ground receiver,

$$\eta_p^2 = \frac{\frac{2T}{K}}{\frac{N_o}{P} + (1 - \frac{1}{K}) \frac{1}{2W}} \quad (A-22c)$$

This expression is valid for all correlation receivers. The detection procedure will be some function of this quantity.

If  $d$  is the duty factor of the pulsed pseudo-noise signals then the total downlink average power is

$$dK P_K = P \quad (A-23)$$

By using the same reasoning used to obtain equation (A-22) we have

$$\eta_p^2 = \frac{4WTP_K}{2N_o W + d(K-1)P_K} \quad (A-24a)$$

From equation (A-23) and (A-24a), we have

$$\eta_p^2 = \frac{\frac{2T}{dK}}{\frac{N_o}{P} + (1 - \frac{1}{K}) \frac{1}{2W}} = \frac{\frac{2T_S}{K}}{\frac{N_o}{P} + (1 - \frac{1}{K}) \frac{1}{2W}} \quad (A-24b)$$

where  $T_S$  is the sampling period, and  $T = dT_S$ . Equation (A-24b) shows that the pulsed pseudo-noise signals behave much like a CW pseudo-noise signal as far as signal-to-noise ratio is concerned. This is true provided the average clutter approximates CW clutter.

This behavior can be understood from the following physical argument. If  $T_S$  is the sampling rate and pulsed pseudo-noise is used of duration  $dT_S$ , the energy per decision is apparently reduced. However, on the average, the percentage number of

signals that use the satellite repeater during the same time interval as the desired signal is also  $d$  and therefore the power per signal in the downlink is for all practical purposes  $P/(dK)$  where  $dK$  is the average number of signals that are active. The energy per decision is, simply,  $E = dT_S P/(dK) = (PT_S)/K$ . It is expected that the deviation from this ideal will be very small when  $d \geq 10\%$  and will increase somewhat when  $d < 10\%$ , although a sharp decrease is not expected as long as the correlator output error probability law holds.

By manipulating equation (A-24b) we obtain the form

$$\eta^2 = \eta_P^2 / 2 = \frac{P}{\frac{K}{T} \left[ \frac{P(1-1/K)}{2W} + N_o \right]} = \frac{P}{N_t} \quad (A-25)$$

where

$$N_t = \frac{K}{T} \left[ \frac{P(1-1/K)}{2W} + N_o \right] = K(N_{to}/T) \quad (A-26)$$

and

$$N_{to} = N_{co} + N_o = \text{clutter energy} + \text{thermal noise energy}$$

Equation (A-26) is equivalent to a system using a narrow band signal pulse of duration  $T$ , and power  $P$  and of additive thermal noise

power  $(KN_{t0}/T)$  in the equivalent narrow band channel. We can now replace  $P$  by  $P_S$ , the equivalent full load sinusoid power.

Therefore

$$\eta^2 = P_S / N_t \quad (A-27)$$

If the signals have an arbitrary duty (or activity) factor  $d \leq 1$ , then

$$\eta^2 = P_S / (dN_t) \quad (A-28)$$

In order to obtain this expression in terms of the reference SSB-FDM voice system, we write

$$\eta^2 = (1/d) (P_S / N_K) \cdot (N_K / N_t) \quad (A-29)$$

where,

$$N_K = KN_o W_o, \quad (A-30)$$

$$W_o = 4,000 \text{ cps}$$

Equation (A-30) is the noise power in a  $K$  channel conventional voice system. Then from equations (A-26) and (A-30), we have

$$\frac{N_K}{N_t} = \frac{N_K \cdot 2WT}{K [P_S(1-1/K) + 2WN_o]} = \frac{2W W_o T}{\frac{P_S}{N_o} (1-1/K) + 2W} \quad (A-31)$$

$$= \frac{W_o T}{\frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1} \quad (A-32)$$

From equations (A-29) and (A-32) we have

$$\gamma^2 = \left( \frac{P_S}{dN_K} \right) \frac{W_o T}{\frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1} \quad (A-33)$$

Substituting equation (A-33) into equation (A-16), gives

$$\begin{aligned} (T.T./N)_{total} = & 2.2 \left( \frac{P_S}{dN_K} \right) \frac{W_o T}{\frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1} - 10 \log_{10}(M-1) \\ & - 10 \log_{10} \left\{ 1 + \frac{Q(M)}{(M-1)} \exp \left[ \frac{1}{2} \left( \frac{P_S}{dN_K} \right) \frac{W_o T}{\frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1} \right] \right\}^{-6} \end{aligned} \quad (A-34)$$

When  $2W \rightarrow \infty$ , we obtain the thermal noise limited case,

$$\begin{aligned} (T.T./N)_{\infty} = & 2.2 \left( \frac{P_S}{dN_K} \right) W_o T - 10 \log_{10}(M-1) \\ & - 10 \log_{10} \left[ 1 + \frac{Q(M)}{(M-1)} \exp \left( \frac{P_S}{2dN_K} \right) W_o T \right] - 6 \end{aligned} \quad (A-35)$$

This is the ideal performance which would be obtained if the signals had zero mutual interference.

Substituting equations (A-12) and (A-13) into equation (A-34) gives

$$\begin{aligned}
 (T.T./N) = 2.2 \left( \frac{P_S}{dN_K} \right) \frac{1}{\left( \frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1 \right)^{2q}} - 10 \log_{10}(M-1) \\
 - 10 \log_{10} \left\{ 1 + \frac{M^{-2q}}{(M-1)} \left[ \exp 1/2 \left( \frac{P_S}{dN_K} \right) \frac{1}{\left[ \frac{P_S}{N_K} \frac{W_o}{2W} (K-1) + 1 \right]^{2q}} \right] \right\}^{-6}
 \end{aligned}
 \tag{A-36}$$

When  $W \rightarrow \infty$

$$\begin{aligned}
 (T.T./N)_{\infty} = 2.2 \left( \frac{P_S}{dN_K} \right) \frac{1}{2q} - 10 \log_{10}(M-1) \\
 - 10 \log_{10} \left[ 1 + \frac{M^{-2q}}{(M-1)} \exp \left( 1/2 \frac{P_S}{dN_K} \frac{1}{2q} \right) \right]^{-6}
 \end{aligned}
 \tag{A-37}$$

Assume that the quantization noise is small relative to the decision noise. Then,

$$(T.T./N) = 2.2 \left( \frac{P_S}{dN_K} \right) 1/(2q) - 10 \log_{10}(M-1) - 6 \tag{A-38}$$

When  $d = 1/2$ ,  $m = 1$ ,  $M = 2$

$$(T.T./N)_{\infty} = 2.2 \left( \frac{P_S}{N_K} \right) 1/q - 6 \tag{A-39}$$

Equation (A-39) is the result for on-off binary transmission using bit-by-bit decision where  $q$  is the number of bits per sample.

When  $q = 1$ ,  $M = 2^m \gg 1$ , we obtain  $(T.T./N)$  for the  $M$ -ary alphabet, for example, one word decision per sample.

Here,

$$(T.T./N)_{\infty} = 1.1 \left( \frac{P_S}{dN_K} \right) - 3m - 6 \quad (A-40)$$

(  $m$  = number of message bits per voice sample. )

#### A. 4 (T. T. /N) At the "Knee" of Input-Output Characteristic

In order to obtain (T. T. /N) at the knee we simply equate the quantization and thermal noise in equation (A-36). Then

$$\frac{M^{-2q}}{(M-1)} \exp \left\{ \frac{1}{2} \left( \frac{P_S}{dN_K} \right) \frac{1}{\left( \frac{P_S}{N_K} \frac{W_o}{2W} (K-1) + 1 \right)^{2q}} \right\} = 1 \quad (A-41)$$

Substituting equation (A-41) back into equation (A-36), gives the expression for the test tone-to-total noise ratio at the knee,

$$(T. T. /N)_t = 3 (2mq - 3) \quad (A-42)$$

In the thermal noise limited case

$$\left( \frac{P_S}{N_K} \right)_{\infty} = 4qd \left[ 2q \log M + \log (M-1) \right] \quad (A-43)$$

(Note, the above is not in db.)

Equation (A-42) is graphed in Figure 1 as a function of  $q$  with  $m = \log_2 M$  as a parameter. Equation (A-43) is graphed in Figure 2 as a function of  $q$  with  $m$  as a parameter. The relationship of  $(T. T. /N)_{\infty}$  in db as a function of  $\left( \frac{P_S}{N_K} \right)_{\infty}$  in db is graphed in Figure 3. For a given parameter  $m$  the pair  $\left\{ (T. T. /N)_{\infty}, \left( \frac{P_S}{N_K} \right)_{\infty} \right\}$  are obtained for the same value

of  $q$ . It should be recognized that the curves plotted are the envelopes of the knees at the threshold.

For on-off binary the clutter is reduced by a factor of two leading to a 3 db decrease in the required intrinsic signal-to-noise ratio. The number of channels per megacycle is doubled over that obtained by using orthogonal binary.

If bi-phase modulation is used RF phase coherent detection is required. This too will lead to a 3 db decrease in the required intrinsic signal-to-noise ratio; the number of channels per megacycle will also be doubled.

In general, if  $\lambda$  is the cross correlation coefficient among the symbols, and if RF phase coherent detection is used, the right side of equation (A-43) will be multiplied by  $\frac{1}{(1 - \lambda)}$ . When

$\lambda = -1$ , we have bi-phase modulation. When  $\lambda = 0$ , we have the orthogonal case while  $\lambda > 0$  indicates positive correlation and requires an increase in  $\left(\frac{P_S}{N_K}\right)_\infty$ . If post detection decisions are used, the non-orthogonal operation requires multiplication of the right side of equation (A-43) by  $\frac{1}{(1 - |\lambda|)}$ . (This appears to be a reasonable approximation.)

#### A.5 Computation of $(P_S/N_K)$ Versus Channels per Megacycle for Given $(T.T./N)$

We will now develop a computational procedure for calculating  $P_S/N_K$  versus the number of voice channels per megacycle for a given  $(T.T./N)$ . In order to do this, we compute  $(T.T./N)_\infty$  for the clutterless case. We then narrow the RF band, add clutter, at the same time increasing  $(P_S/N_K)$  so that the  $(T.T./N)_\infty$  is maintained. As more clutter is added, (i.e., more active users) the signal-to-thermal noise ratio can be increased so as to maintain a constant  $(T.T./N)$ . If this process is continued in the limit, the channel becomes clutter limited at the point when the clutter channel capacity equals the thermal noise channel capacity required to maintain  $(T.T./N)_\infty$ .

If equation (A-41) is solved for  $(P_S/N_K)$ , we obtain

$$\frac{P_S}{N_K} = \frac{4qd [2q \log M + \log (M-1)]}{1 - \frac{W_o}{2W} (K-1) [2q \log M + \log (M-1)] 4 qd} \quad (A-44)$$

It is clear from equation (A-43) and equation (A-44) that for the same parameters

$$\frac{P_S}{N_K} = \frac{\left(\frac{P_S}{N_K}\right)_\infty}{1 - \frac{W_o}{2W} (K-1) \left(\frac{P_S}{N_K}\right)_\infty} \quad (A-45)$$

Let,

$$Q = 10 \log_{10} (P_S/N_K) - 10 \log_{10} (P_S/N_K)_{\infty} = 10 \log_{10} \left[ \frac{\frac{2W}{W_o(K-1)(P_S/N_K)_{\infty}}}{\frac{2W}{W_o(K-1)(P_S/N_K)_{\infty}}^{-1}} \right] \quad (A-46a)$$

where  $Q$  is the intrinsic signal-to-noise ratio multiplexing penalty, in db. For computational purposes let,

$$Q(X) = 10 \log_{10} \left( \frac{X}{X-1} \right) \quad (A-46b)$$

where

$$X = \frac{2W}{W_o(K-1)(P_S/N_K)_{\infty}} \quad (A-46c)$$

The function  $X$  is dependent on the channel parameters as seen in equation (A-46c). The function  $Q(X)$  is shown in Figure 4.

For a given  $(T.T./N)_{\infty}$ , we obtain the corresponding  $(P_S/N_K)_{\infty}$  in db. Thus, the function  $Q(X)$  is applicable to all types of signal alphabets. Assume  $K \gg 1$ , which is true in practice. Then

$$X = \frac{250}{G(P_S/N_K)_{\infty}} \quad ; \text{ (Note } P_S/N_K \text{ is not in DB)} \quad (A-47)$$

where  $G = K/2W$  = channels per megacycle, and

$\frac{10^6}{W_o} = \frac{10^6}{4,000} = 250$ , is the maximum number of channels per megacycle attainable, i. e., by using an SSB-FDM voice system.

Thus,

$$10 \log_{10} \left( \frac{P_S}{N_K} \right) = 10 \log_{10} \left( \frac{P_S}{N_K} \right)_{\infty} + Q(X) \quad (A-48)$$

Then, from equation (A-47),

$$G(X) = \frac{250}{X (P_S/N_K)_{\infty}} \quad (A-49)$$

There is a  $(T.T./N)_{\infty}$  corresponding to  $(P_S/N_K)_{\infty}$  which is held constant. This computation results in the family of curves shown in Figures (5) and (6).

We can now rewrite  $Q$  as

$$Q(G) = 10 \log_{10} \left[ \frac{250}{250 - G(P_S/N_K)_{\infty}} \right] \quad (A-50)$$

Referring back to equation (A-46c) it is seen that when

$X = 1$ ,  $Q = \infty$ . At this point the channel becomes clutter limited. Here we have

$$\frac{2W}{(K-1)} = W_o \left( \frac{P_S}{N_K} \right)_{\infty} \quad (A-51)$$

If we note that

$$\left( \frac{P_S}{N_K} \right)_{\infty} = \frac{P_S}{N_o K W_o}$$

then,

$$\frac{2W}{(K-1)} = \frac{P_S}{N_o K} = \frac{C_\infty}{K} \quad (A-52)$$

For  $K \gg 1$ ,  $\frac{2W}{K}$  is a parameter which we will call the clutter limited "channel capacity" per message while  $\frac{C_\infty}{K}$  is the thermal noise capacity per message in the absence of clutter. In order to maintain a constant (T. T. /N), i. e., a constant performance, it is essential to maintain a constant channel capacity.

An examination of equation (A-49) shows that the number of channels per megacycle is inversely proportional to  $\left(\frac{P_S}{N_K}\right)_\infty$  (not in db). Thus a 3 db increase in  $\left(\frac{P_S}{N_K}\right)_\infty$  will halve the number of channels per megacycle. In order to explore this relationship further, let us substitute equation (A-43) into equation (A-49). Then,

$$G(X) = \frac{63.5}{X \, dq [2q \log_e M + \log_e (M-1)]} \quad (A-53)$$

From equation (A-53), it is clear that the number of channels per megacycle is inversely proportional to the activity factor  $d$ . In conventional binary transmission,  $M = 2$ , then

$$G_q(X) = \frac{45}{X \cdot dq^2} \quad (A-54)$$

On the other hand for the single word binary case  $q = 1$ ; assume  $M \gg 1$ , then,

$$G_M(X) \approx \frac{2l}{X d \log_e M} \quad (A-55)$$

The efficiency in channels per megacycle of a conventional binary system to M-ary system both of which use PN-multiplexing is

$$E_g = \frac{G_q(X)}{G_M(X)} = \frac{1.5 m}{q^2} \quad (A-56)$$

where  $M = \exp_2 m \gg 1$ . When  $m = q$ , the efficiency varies as  $1/q$ . Thus, for high-quality systems, the M-ary single word decision technique is far more efficient than the bit-by-bit decision procedure.

#### A.6 A Utility Function for Choosing a PN-System Operating Point

It is clear from the curves shown in Figure 6 that there are many acceptable operating points for a given voice quality. Where power and bandwidth are at a premium a convenient operating point may be chosen by defining a utility function

$$U = (P_S/N_K)_{\text{db}} \cdot 1/G \quad \text{for a given } (T.T./N) \quad (\text{A-57})$$

Thus,  $U$  is directly proportional to the product of power and the bandwidth per talker. We can therefore graph  $U$  versus

$(P_S/N_K)_{\text{db}}$  as shown in Figure 7 and operate at the point

$(P_S/N_K)_{\text{db}}$  where  $U$  is minimum. This utility function exchanges bandwidth for  $(P_S/N_K)$  in db.

Where downlink power is at a premium and not RF bandwidth this utility function is not representative of this situation. In this case, however, the system would be designed such that performance is thermal noise limited, i. e.,  $W \gg \frac{P_S}{N_o}$ .

## APPENDIX B

### PN-Multiplexing Systems Using Analog FM Signaling Techniques

#### 1.1 Mathematical Analysis

In order to develop the theory of PN-multiplexing for voice signals which frequency modulate a sinusoidal sub-carrier, it is convenient to use a mathematical model derived in Ref. 3. In this model the audio output signal-to-noise ratio can be calculated along with the threshold characteristic which approximates that which would be obtained with feedback. An interesting property of this model is that it postulates an M-ary decision procedure for locating the filter which contains the desired signal which is then converted to an analog voltage by an FM discriminator. The threshold characteristic of this model which approximates FM-FB is strongly influenced by the M-ary decision error probability, which was used in appendix A for the study of digital techniques. Thus the theory previously developed closely resembles the FM model used here. We will now develop the theory of pseudo-voice FM using the mathematical model developed in Ref. 3.

The output noise power due to incoming thermal noise is given by

$$\sigma_n^2 = \frac{1}{12\eta_p^2} = \frac{1}{24\eta^2} \quad (\text{B-1})$$

where,

$$2\eta^2 = \eta_p^2 = \frac{P_S}{dN_K} \frac{1}{(P_s/N_K) [W_o/(2W)] (K-1) + 1} \quad (\text{B-2})$$

The decision error noise which essentially determines threshold behavior is

$$\sigma_n^2 = \frac{\mu(\mu+1)(\mu+2)\alpha}{12} \quad (\text{B-3})$$

The audio signal power is given by

$$S^2 = 1/2 (\mu/2)^2 [1 - (\mu+1)\alpha]^2 \quad (\text{B-4})$$

The audio-output signal-to-noise ratio is given by

$$(S/N)_o = 3/2 \mu^2 \eta_p^2 \frac{[1 - (\mu+1)\alpha]^2}{1 + \mu(\mu+1)(\mu+2)\alpha\eta_p^2} \quad (\text{B-5})$$

The knee of the threshold characteristic is obtained by equating the competing noises. Hence

$$\alpha = \frac{1}{2\eta^2 \mu(\mu+1)(\mu+2)} \quad (\text{B-6})$$

where,

$$\alpha = (\mu/2) \exp(-1/2\eta^2) \quad (\text{B-7})$$

(That is, the FM index  $\mu$  is equal to  $(M+1)$  when  $M$  represents the number of filters used in the model). At this point  $(S/N)_0$  is for all practical purposes

$$(S/N)_0 = 3/2\mu^2\eta^2 \quad (\text{B-8})$$

At the threshold

$$\exp(-1/2\eta^2) = \eta^2\mu^2(\mu+1)(\mu+2) \quad (\text{B-9})$$

Let  $\eta_\mu^2$  be the input signal-to-noise ratio at the threshold of the FMFB receiver, obtained from Equation (B-9). From Equation (B-2),

$$\left(\frac{P_S}{N_K}\right) = \frac{2d\eta_\mu^2}{1 - \frac{W_0}{2W}(K-1)2d\eta_\mu^2} \quad (\text{B-10})$$

When  $W \rightarrow \infty$ , we obtain the threshold signal-to-noise ratio when performance is thermal noise limited only. Then from Eq (B-10)

$$\left(\frac{P_S}{N_K}\right)_\infty = 2d\eta_\mu^2 \quad (\text{B-11})$$

As in appendix A, for digital transmission, we have,

$$\left(\frac{P_S}{N_K}\right) = \frac{(P_S/N_K)_\infty}{1 - \frac{W_0}{2W} (K-1) (P_S/N_K)_\infty} \quad (B-12)$$

The PN-FMFB modulation technique is similar to a higher-order signal alphabet using multiple FSK. The comparison theory for digital communication developed in Appendix A assumes binary sequences. (The general theory developed in Ref. (1) treats multiple FSK as well.) In the case of multiple FSK, the clutter generated will be less than in the case of random binary sequences. (20) It is quite simple to modify the basic relationships simply by including a clutter factor  $0 < c \leq 1$  which modifies the clutter term in Equation B-2. Thus,

$$\eta_p^2 = P_S / (dN_K) \frac{1}{\frac{P_S}{N_K} \left( \frac{cW_0}{2W} \right)^{(K-1)+1}} \quad (B-13)$$

It seems reasonable to postulate a value of  $c = .50$ .

In addition to the clutter factor it is also essential to derive a modified expression for the number of channels per megacycle. This too follows from the model for FMFB used here. The total bandwidth for PN-FSK is given by

$$W_t = 2W + 2(\mu + 1)W_o = 2W + W_{FM} \quad (B-14)$$

where  $2W$  is the bandwidth of a single frequency shifted pseudo-noise signal. In order to obtain the number of channels per megacycle, we divide Eq (B-14) by  $KW_o$  and obtain

$$\frac{250}{G_t} = \frac{250}{G} + \frac{250}{G_\mu}$$

or

$$G_t = \frac{G G_\mu}{G + G_\mu} \quad (B-15)$$

where,

$$G_\mu = 125/(\mu + 1) \quad (B-16)$$

is the number of channels per megacycle in a conventional FM system and

$$G = \frac{250}{cX(P_S/N_K)_\infty} \quad (B-17)$$

is the channels per megacycle obtained for the PN-multiplexing techniques of appendix A.

Equation B-15 shows that when  $G_\mu \gg G$  (i.e., narrow band FM),  $G_t = G$ , or the number of channels per megacycle is determined by the PN subcarrier. When  $G \gg G_\mu$ ,  $G_t = G_\mu$  and behavior is equivalent to conventional multiplexing. (It appears reasonable to constrain  $G$  such that  $G_\mu \gg G$  or equivalently, that the PN-subcarrier bandwidth is greater than the FM bandwidth without the subcarrier.)

The computation procedure for  $(P_S/N_K)$  versus  $G_t$  is the same as before. By using Equations B-15, B-16, we can obtain the performance curves shown in Fig. 6.

Above threshold, the decision error noise can be neglected; performance being limited by thermal noise and clutter. Then

$$(T.T./N) = 10 \log_{10} \frac{P_S}{dN_K} - 10 \log_{10} \left[ \frac{P_S}{N_K} \left( \frac{W_0}{2W} \right)^{c(K-1)+1} \right] + 20 \log_{10} \mu - 4 \quad (B-18)$$

When  $W \rightarrow \infty$ ,

$$(T.T./N)_\infty = 10 \log_{10} \frac{P_S}{dN_K} + 20 \log_{10} \mu - 4 \quad (B-19)$$

Once again, when the PN-multiplexing loss is neglected by letting  $W \Rightarrow \infty$ ,  $(T.T./N)$  takes on the same form as for conventional FM above threshold. Equation B-19, however, contains the pseudo-noise processing gain as well as the FM modulation index gain.

The additional constraint is that for a given FM modulation index the intrinsic signal-to-noise ratio must be above the threshold so that Equation B-18 holds. This can easily be checked by using Equation B-10.

## APPENDIX C

### Summary of Significant Results in PN-Multiplexing Techniques

#### C.1 Digital Communications

Test tone-to-total noise ratio at the knee of the threshold characteristic is

$$(T.T./N)_t = 3 (2mq - 3) \quad (C-1)$$

$(P_S/N_K)$  when bandwidth is infinite, i.e.,  $(P_S/N_K)_\infty$  is given by

$$(P_S/N_K)_\infty = 4 \text{ qd} \left[ 2q \log M + \log (M-1) \right]; \text{ (not in DB)} \quad (C-2)$$

Finite bandwidth  $P_S/N_K$  in terms of  $(P_S/N_K)_\infty$

$$10 \log_{10} (P_S/N_K) = 10 \log_{10} (P_S/N_K)_\infty + Q(X) \text{ DB} \quad (C-3)$$

The function  $Q(X)$  (PN-multiplexing penalty) is

$$Q(X) = 10 \log_{10} X/(X-1) \text{ DB} \quad (C-4)$$

and channels per megacycle is

$$G(X) = \frac{250}{X(P_S/N_K)_\infty} \quad (C-5)$$

### Computational Procedure

- a. Graph  $(T. T. / N)_{\infty}$  as a function of  $q$  with  $m$  a parameter;  
 $m = \log_2 M$  (Equation C-1)
- b. Graph  $(P_S / N_K)_{\infty}$  as a function of  $q$  with  $m = \log_2 M$  a  
parameter (Equation C-2)
- c. Graph  $(T. T. / N)_{\infty}$  versus  $(P_S / N_K)_{\infty}$  with  $m$  as a para-  
meter
- d. Graph universal curve  $Q(X)$  Equation C-4
- e. Compute  $10 \log_{10}(P_S / N_K)$  from Equation C-3 for a  
given value of  $X$ .
- f. Compute  $G(X)$  for same value of  $X$  and  $(P_S / N_K)_{\infty}$   
(Equation C-5)
- g. From e. and f. a point on  $(P_S / N_K)$  versus  $G$  is obtained  
for  $(T. T. / N) = (T. T. / N)_{\infty}$

The general expression for  $(T. T. / N)$  is given by;

$$(T.T./N) = 2.2 \frac{P_S}{dN_K} \left[ \frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1 \right]^{-10 \log_{10}(M-1)} - 10 \log_{10} \left[ 1 + \frac{M^{-2q}}{(M-1)} \exp \left( \frac{1}{2} \frac{P_S}{dN_K} \left[ \frac{P_S}{N_K} \left( \frac{W_o}{2W} \right) (K-1) + 1 \right]^{2q} \right) \right]^{(C-6)} \quad -6$$

## 2. PN-FM (Above Threshold)

Obtain (T.T./N) when  $W \rightarrow \infty$

$$(T.T./N)_{FM\infty} = 10 \log_{10} \frac{P_S}{dN_K} + 20 \log_{10} \mu - 4 \quad (C-7)$$

$$G_t = \frac{G G_\mu}{G + G_\mu} \quad (C-8)$$

where,

$$G = \frac{250}{c X \left( \frac{P_S}{N_K} \right)^\infty} \quad 0 < c \leq 1 \quad (C-9)$$

and,

$$G_\mu = \frac{125}{\mu + 1} \quad (C-10)$$

## Computational Procedure

- Graph Equation C-7
- Follow procedure e., f., g., for digital case

## General Result

$$(T.T./N)_{FM} = 10 \log_{10} \frac{P_S}{dN_K} - 10 \log_{10} \left[ \frac{P_S}{N_K} \frac{cW_o}{2W} (K-1) + 1 \right] + 20 \log_{10} \mu - 4 \quad (C-11)$$

At the knee of the threshold characteristics

$$(1/d) \left( \frac{P_S}{N_K} \right)_{\infty} = 2 \eta_{\mu}^2 \quad (C-12)$$

and

$$(T.T./N)_{\infty} = 10 \log \eta_{\mu}^2 + 20 \log \mu - 7 \quad (C-13)$$

## APPENDIX D

### Summary of Performance

#### Relationships of Conventional Modulation Techniques

This work is a summary of the results in (2) and is summarized here for the sake of completeness

##### 4.1 $\left(\frac{T.T.}{N}\right)$ for SSB

When number of channels is  $K \geq 240$

$$\left(\frac{T.T.}{N}\right) = 10 \log_{10} \left(\frac{P_S}{N_K}\right) + 9 \quad (D-1)$$

When number of channels is  $K < 240$

$$\left(\frac{T.T.}{N}\right) = 10 \log_{10} \left(\frac{P_S}{N_K}\right) - 5 + 6 \log_{10} K \quad (D-2)$$

##### 4.2 $\left(\frac{T.T.}{N}\right)$ for FM

When number of channels is  $K \geq 240$

$$\left(\frac{T.T.}{N}\right) = 10 \log_{10} \left(\frac{P_S}{N_K}\right) + 20 \log_{10} \mu + 10.8 \quad (D-3a)$$

where

$$\mu = \frac{\Delta f}{K W_o} \quad (D-3b)$$

and

$\Delta f$  = peak deviation

$\mu$  = index

In addition, we have the relationship

$$W_{FM} = 2 (\mu + 1) W_o K \quad (D-4a)$$

Let  $K/W_{FM} = G$  - number of channels per megacycle. Then,

$$G = \frac{125}{\mu + 1} \quad (D-4b)$$

(These results assume that the FDM signal occupies a low pass bandwidth.)

When number of channels is  $K < 240$

$$\left( \frac{T.T.}{N} \right) = 10 \log_{10} \left( \frac{P_S}{N_K} \right) - 3.2 + 6 \log_{10} K + 20 \log_{10} \mu \quad (D-5)$$

### 4.3 Frequency Division Multiplex Multi-carrier Phase Modulation

For a 1% time overload, we have

$$\left(\frac{T.T.}{N}\right) = 10 \log_{10} \left(\frac{P_S}{N_K}\right) - 8.5 + 6 \log_{10} K_c + 20 \log_{10} \mu \quad (D-6a)$$

$K_c$  = number of carriers

The channels per megacycle is half that of Equation D.4b. Hence,

$$G = \frac{62.5}{\mu + 1} \quad (D-6b)$$

### 4.4 Conventional PCM

Interleaved PCM (PCM-AM on-off)

$$\left(\frac{T.T.}{N}\right) = 2.2 \left(\frac{P_S}{N_K}\right) (1/q) - 6 \quad (D-7)$$

Channels per megacycle for PCM

The duration per bit is

$$T = T_S / (qK) = 1/(2W) \quad (D-8a)$$

$$T_S = 1/(2W_o)$$

Then

$$G = \frac{125}{q} \quad (D-8b)$$

Multi-carrier PCM Phase Reversal Modulation (0.1% of time  
overload)

$$\left(\frac{T.T.}{N}\right) = 0.8 \left(\frac{P_S}{N_K}\right)^{1/q - 6} \quad (D-9)$$

Since the bandwidth here is doubled, we have

$$G = \frac{62.5}{q} \quad (D-10)$$

## Appendix E

### THRESHOLD M-ARY DETECTION

Threshold M-ary detection differs from greatest-of detection in that a single threshold is set across the outputs of an array of correlation receivers. A correct decision is indicated if the filter output which contains the desired signal exceeds the threshold while all other filter outputs do not. This type of operation is of significant practical interest when the type of modulation used is some form of pulse-time modulation (i. e., pulse-position, pulse-rate, pulse width, etc). Where natural pulse-rate modulation is used, it permits asynchronous operation with a theoretical improvement factor equal to that obtained with FM.

For M-ary alphabets of large size, equivalent behaviour is obtained for coherent and incoherent detection. In addition it can be shown <sup>(16), (17)</sup> that channel capacity can be realized in the threshold case just as for the case of the greatest-of decision procedure.

From Reference 16 using our symbols, the error prob-

ability is given by

$$\alpha = (M-1) \int_{r\eta_p}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} y^2) dy + \int_{(1-r)\eta_p}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} y^2) dy \quad (E-1)$$

where  $\eta_p$  is given in Equation (B-13) with  $c=1$  and  $r$  is the ratio of the threshold to the peak signal amplitude. The threshold which minimizes the error probability is given by

$$r = \frac{1}{2} + \frac{\log(M-1)}{\eta_p^2} \quad (E-2)$$

By introducing further approximations to (E-1) and using the methods in Appendix A, it can be shown that the knee in the characteristic when digital transmission is used is given by

$$\frac{1}{2} \eta_p^2 = \log(M-1) + 2 \log \frac{1}{\alpha} + 2 \sqrt{\log 1/\alpha} \sqrt{\log(M-1) + \log 1/\alpha} \quad (E-3)$$

For  $M \gg 1$ ,

$$\eta^2 = 10 \log M + 2 \log 2. \quad (E-4)$$

When a greatest-of decision is used we have, for  $M \gg 1$ ,

$$\eta^2 = 6 \log M \quad (E-5)$$

Thus, the simple threshold  $M$ -ary decision case is only 2db worse than the more complex greatest-of decision procedure. These results can be put in the standard form of Appendix A by using the same methods.

PPM or pulse-rate modulation is a simple type of modulation technique which exchanges power for bandwidth the same as FM, giving the same improvement. However, these time modulations require extremely large peak-power since for wide deviation the pulse must be very short. This is perhaps the reason why this modulation has received very little attention in satellite applications since, here, the repeaters are peak-power limited. However, pulsed pseudo-noise communications using matched filter reception achieves the improvements of wide deviation PPM since the autocorrelation function is a pulse of short duration and at the same time the peak-power requirements are reduced by using a long pulse. Where  $P_S/N_0$  is at a premium, and not bandwidth, it is likely that the type of exchange of  $P_S/N_0$  for bandwidth which wide deviation PPM affords may be much simpler and less expensive than FMFB. Since FMFB (3) and PPM<sup>(15), (16)</sup> both yield M-ary behavior there is little to choose between the two on a theoretical basis.

The threshold behavior in analog PPM can be obtained by using the same model as in Reference (3) for FMFB. Above threshold ( $T.T./N$ ) is identical to that given in Equation (C-10), for FMFB where  $\mu$  is the ratio of time deviation to the duration of the correlation peak.

## Appendix F

### F-T MATRIX TECHNIQUES

#### F.1 Introduction

In this appendix a particular form of pulse-addressing, which has found some interest in random access communications applied to tactical army communications, is discussed. The performance of this technique with respect to high quality communications over a satellite link is evaluated. The major interest here is to familiarize the reader with the performance of pulse addressing techniques, of which the one discussed here is typical. As a possible modulation technique for multiple access satellite communications of good voice quality, it is inadequate.

An FT-matrix technique can assume many forms. The curves presented in this report cover only one such form.

The technique selected for analysis and comparison is the use of a gated FT receiver which uses delta modulation (DM). This method was chosen for two reasons:

- (1) IBM has performed extensive analyses and optimizations of this technique for the Army on a sub-contract to Motorola.
- (2) It has significant advantages over other FT techniques.

The DM-FT approach performs its modulation in two steps. First, the voice is converted to binary digits by the delta modulator. Such modulators have been described extensively in the literature.

Secondly, the binary data is converted into an FT matrix pattern. This conversion is described in depth in the Final Design Plan submitted by Motorola to the Army. The reader can refer to this document for an explanation of the system and for the development of some of the results to be presented below.

In order to evaluate DM-FT, it is necessary to relate audio S/N to binary error rate in DM. There are two sources of audio noise using DM. The first is the noise power due

to quantizing,  $N_Q$ . The second is the noise power due to binary errors,  $N_E$ .

## F.2 Theoretical-Signal-to-Noise Ratio in Single Integration Delta Modulators

Definition of symbols:

$S$	=	power in test sinusoid at output of delta demodulator
$f_B$	=	samples per second in audio channel
$f_H$	=	high frequency cut-off of DM demod filter
$f_L$	=	low frequency cut-off of DM demod filter
$\sigma$	=	height in volts of one DM step
$f_s$	=	frequency of test sinusoid
$N_E$	=	noise power at demodulator output due to binary errors
$N_Q$	=	noise power at demodulator output due to quantizing
$P_e$	=	binary error rate.

#### Error Noise Power:

Each error introduces a step function of  $2\sigma$  volts into the final audio bandpass filter. The average power introduced by these noise step functions at the output of the bandpass filter is

$$N_E = p_e f_B \int_{2\pi f_L}^{2\pi f_H} (2\sigma/\omega)^2 d\omega \quad (F-1)$$

$$N_E = \frac{2\sigma^2}{\pi} p_e f_B \left\{ \frac{1}{f_L} - \frac{1}{f_H} \right\} \quad (F-2)$$

$$N_E \doteq \frac{2\sigma^2 p_e f_B}{\pi f_L} \quad (F-3)$$

#### Quantizing Noise Power:

Under the assumption that the quantizing noise is uniformly distributed over a range of  $\sigma$  volts, that it is independent from DM sample to sample, and that the power is uniformly distributed from 0 to  $f_B/2$ , we write,

$$N_Q \doteq \frac{\sigma^2}{12} \left\{ \frac{f_H - f_L}{f_B/2} \right\} = \frac{\sigma^2 f_H}{6f_B} \quad (F-4)$$

Output Signal Power:

The output of the saturated system is a triangular wave with peak to peak amplitude of  $\sigma_B/2f_s$ .

The power of the first harmonic is

$$S = 2\sigma_B^2/\pi^4 f_s^2 \quad (F-5)$$

Signal to noise ratio:

Assuming that quantizing and error noise power are additive, it follows from Eqs. (F-3), (F-4), and (F-5) that,

$$\frac{S}{N_E + N_Q} = \frac{12f_D^3 f_L}{f_H f_L \pi^4 f_s^2 + 12\pi^3 p_e f_B^2 f_s^2} \quad (F-6)$$

In the previous paragraphs, it is shown that for a sinusoid of frequency  $f_s$  and of sufficient amplitude to saturate the system the signal-to-noise ratio is given by

$$\frac{S}{N_E + N_Q} = \frac{12f_B^3 f_L}{f_H f_L \pi^4 f_s^2 + 12\pi^3 p_e f_B^2 f_s^2} \quad (F-7)$$

One other item of information is required for the complete evaluation. This is the relation of  $P_e$ , the binary error rate, to the system loading. Deriving such a result is not a simple chore.

This has been done in the report referred to previously. Both simulation and analysis were used to arrive at the approximate relation,

$$P_e = \frac{Kf_B}{W} (1.63)(10^{-7}) \quad (F-8)$$

where

$K$  is the number of active users (i.e., talkers)

$W$  is the bandwidth in megacycles

$f_B$  is the binary data rate.

Substituting Equation (4) into Equation (3) yields,

$$\frac{S}{N_E + N_Q} = \frac{12 f_B^3 f_L W}{W f_H f_L f_s^2 \pi^4 + 12 \pi^3 (1.63)(10^{-7}) K f_B^3 f_s^2} \quad (F-9)$$

### F.3 Experimental Results

Figure F.1 contains a plot of audio S/N in db versus binary error rate. This is the result of an IBM laboratory measurement. The result of the lab measurement in Figure F.1 corresponded closely with the results predicted in Appendix E.

Figure F.2 contains laboratory measurements which relate quantizing noise to pulse rate for PCM, single integration DM and double integration DM. These curves were measured by de Jager of Phillips. For the single integrator case, they largely confirm Appendix E. Although the curves were produced for a signal at 800 cps extrapolation to 1,000 cps is straightforward using Appendix E.

Figure F.3 indicates the relation of binary error rate to active users per Megacycle for a FT matrix system. This result was derived using both analysis and simulation and is based on the assumption that both desired and undesired signals arrive at the receiver with equal power.

Combining Figures F.1 and F.3 we have obtained Figure F.4. These two curves tell the story as far as FT matrix techniques are concerned. The story is none too encouraging from the point of view of telephone quality. For example, in order to get 40 db audio S/N about 50 mc per talker are needed according to Figure F.4.

Let us take a detailed look at the reasons behind Figure F.4. Figure F.4 indicates that it is very expensive in bandwidth to get telephone quality transmission. The cause of this is shown in Figure F.3 namely, binary error rate is directly proportional to talkers per megacycle. This proportionality is due to the nature of FT addressing.

Each receiver is looking for a pattern of one or more pulses each bit time. If these pulses are detected, a post-detection decision that a "1" was sent is made. If the pulses are not detected, the decision is that a "0" was sent.

There are many variations on this scheme. For example, a "1" could consist of five pulses arranged in a particular FT pattern and the receiver could decide that a "1"

was sent if any three of the pulses are detected. This is the concept of  $m$  of  $n$  (3 of 5 in this case) logic. Analysis and simulation have shown that different values of the pair  $(m, n)$  with properly adjusted decision thresholds, all yield approximately the same results as those appearing in Figure F. 3. The assumptions implicit in Figure F. 3 break down somewhat in the region of error rate greater than 2%, but this is not the region of interest for telephone quality.

When a receiver opens a gate to admit a pulse, it will make an error if its own pulse was not sent but a pulse from an interfering transmitter appears in the gate. Clearly, the probability of this occurring follows essentially a Poisson law. For the probabilities of interest, this implies a linear dependence of error rate upon the number of pulses on the air on the frequency slot to which the gate is tuned. This, in turn, leads to the conclusion that error probability is proportional to active users per megacycle.

The discussion on how a pulse is falsely detected is a little naive in that the possibility of cancelling a true pulse

was not mentioned. This phenomenon was analyzed and simulated in detail, and it was concluded that cancellation, too, results in error rate being directly proportional to active users per megacycle.

One final remark on Figure F.3 is in order. It is assumed therein that all pulses, both desired and undesired, arrive at the receiver at equal levels. Should the interfering pulse power grow, then there occurs a very gradual increase in error rate for a given value of talkers per magacycle.

The second result leading to Figure F.4 is that appearing in Figure F.1. This curve indicates that audio signal-to-noise ratio is proportional to binary error rate. The physical reason for this is evident. Each error causes a noise pulse to enter the delta demodulator. The power of the ensemble of noise pulses is directly proportional to their number and hence to the probability of binary error.

The audio tone for which Figures F.1 and F.3 lead to the result in Figure 4 is not, strictly speaking, a test tone.

The measurements were made for a sinusoid which just saturated the DM demodulator and then was reduced 3 db at the transmitter. Unfortunately, the inherent non-linearities in DM are such that the output power at the test frequency is reduced only 1/2 db when the transmitted tone power is backed off 3 db from saturation. This is one factor that makes the relation to test tone to noise ratio arbitrary.

A second problem is that of companding. Since DM is sensitive primarily to derivative overloading and the distribution of the derivative conventional amplitude companding has an effect on DM entirely different from that on say PCM. On the other hand de Jager is now experimenting with devices which function basically as derivative compandors. There would be a great deal of arbitrariness in incorporating such "companding" into the discussion.

Although, no conversion to a true T. T. /N ratio has been attempted, it can be stated that the curve in Figure 4 are optimistic since they represent an audio tone only 1/2 db below system saturation. Furthermore, the curves are adequate to

demonstrate the inferiority of FT matrix techniques PN methods for high-quality telephone transmission.

There are several more aspects of the FT matrix revealed by Figures 1 to 4 which apparently contradict intuition.

For example, Figure F.2 indicates that signal to quantizing noise ratio increases at a rate of 9 db per octave of sampling rate increase for single integration delta. This is a well known result. However, it is intuitive that the increased sampling rate per channel should require a wider system bandwidth. Surprisingly, this is not the case.

If the system bandwidth is held constant, the per channel  $p_e$  increases linearly with the per channel sampling rate. Since error noise power is directly proportional to  $p_e$  and inversely proportional to sampling rate, the error noise power is unchanged.

On the other hand, Figure F.2 indicates that increasing per channel sampling rate increases signal to quantizing noise ratio. This means that in the FT-DM, the per channel sampling rate can be increased indefinitely, without any increase in bandwidth, reducing quantizing noise to as low a figure as desired.

It follows that FT-DM is strictly error noise limited.

Now there are many experiments on DM variations now in progress. Gains in signal to quantizing noise by as much as 20 db have been obtained. At this time, it appears that such techniques may well make DM competitive with PCM in conventional frequency or time division multiplex. However, since FT matrixing is strictly limited by error noise, these variations are of no help here. In fact, it is usually detrimental to break away from straightforward single integration DM because of increased sensitivity to binary errors.

PCM might appear to have some advantages in an FT system. This too is a misconception. In Figure F.4, PCM and DM are compared. The reasons for the superiority of DM are two fold:

- (1) DM has a lower error noise for a given  $p_e$  than PCM, and the FT matrix system is error noise limited.
- (2) In order to decrease quantizing noise below the error noise it is necessary to increase the sampling rate per channel in both DM and PCM. In DM a greater increase in

rate is required by no increase in total system bandwidth is is needed. On the other hand, the increase in per channel sampling rate in PCM requires a proportional increase in system bandwidth to maintain the signal to error noise ratio.

F.4 The design procedure for the FT matrix system is the following:

- (1) Using Figure 4, determine  $K/W$  for the desired audio  $S/N$ . Certain arbitrary assumptions will have to be made to relate this to a desired  $T. T. /N$ . The system bandwidth is then the product of the number of active users by  $K/W$ .

- (2) Quantizing noise must be reduced below the level of the error noise by increasing the sampling rate per channel. Figure F.5 indicates the required sampling rate per channel using the bandwidth per channel obtained in step (1). It should be borne in mind that increasing per channel sampling rate has no effect on error noise (except for errors introduced by intrinsic channel noise).

Let us summarize the principal conclusions to be drawn from Figures F.1 to F.5.

(1) For telephone quality FT matrixing requires exorbitant bandwidths.

(2) FT matrix systems are error noise limited.

(3) Single integration DM is better than double integration delta or PCM in an FT matrix system because of its superior error noise immunity.

(4) Neglecting thermal noise, the DM sampling rate can be increased without paying a bandwidth penalty in an FT system.

(5) When these results are extrapolated to a system which frequency division multiplexes before delta modulation, the system bandwidth requirements are unchanged.

FT matrixing techniques require exorbitant bandwidths to achieve  $T.T./N$  ratios in the vicinity of 40 and 50 db. They should be rejected for this reason.

PN techniques can attain figures near 10 channel per megacycle as noted above. FT systems yield figures like .01 channel per megacycle.

FT systems have applicability when the received

signal powers from different stations assume a wide range (i.e., the so-called synamic range problem) or when low audio quality (say 13 to 20 db T. T. /N) is acceptable.

(In these calculations the 25% activity factor has not been included since this hardly improves the extremely inefficient bandwidth utilization which characterizes such techniques.)

The use of PN techniques combined with frequency-hopping has many advantages. The results of this Appendix (F) are not to be construed as reflecting adversely on such a combined system. The combined system is entirely different from the FT matrix analyzed above since each pulse in the FT matrix has a WT product of one.

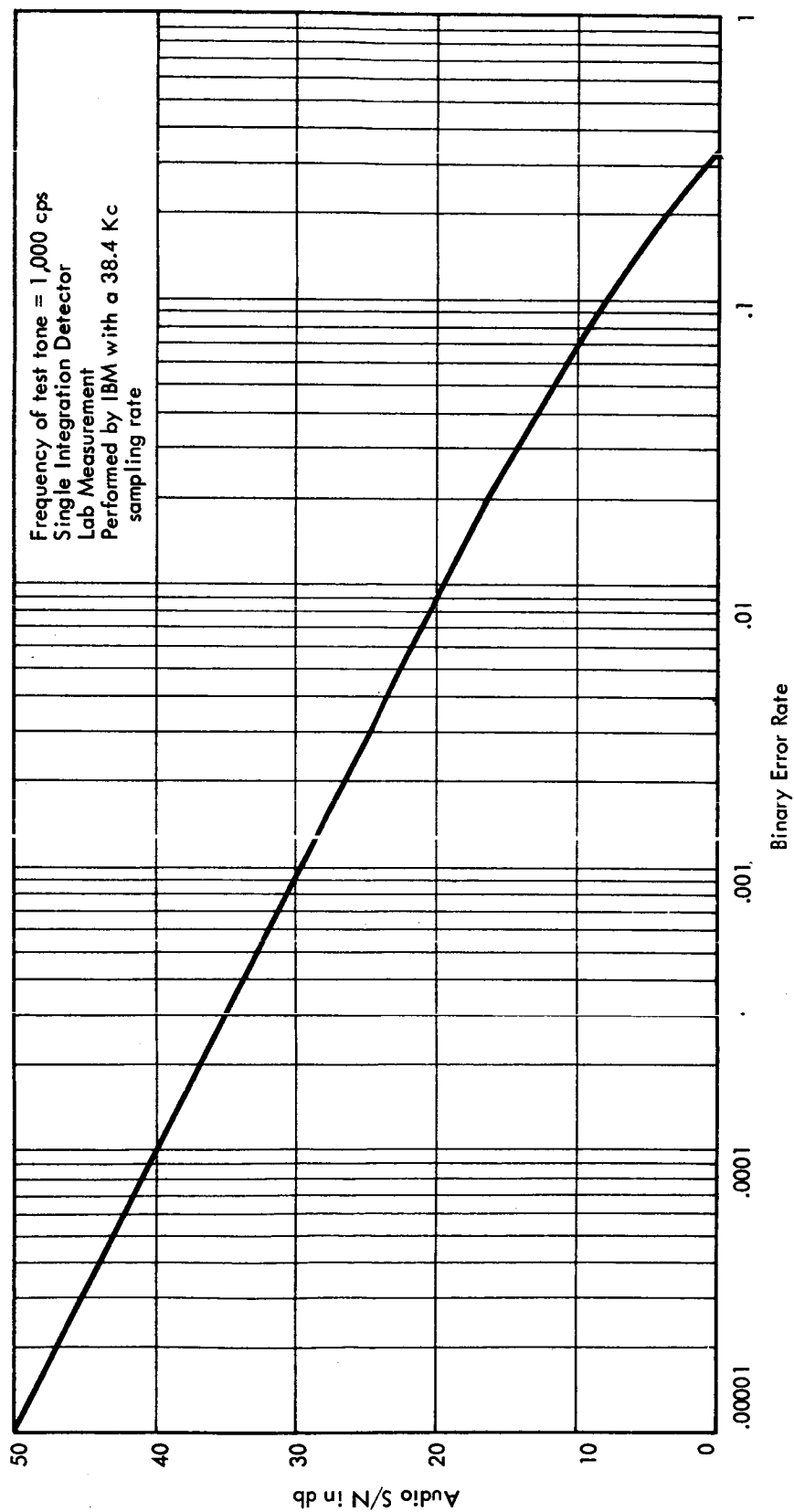


Figure F.1 Signal to DM Error Noise Ratio vs Binary Error Rate

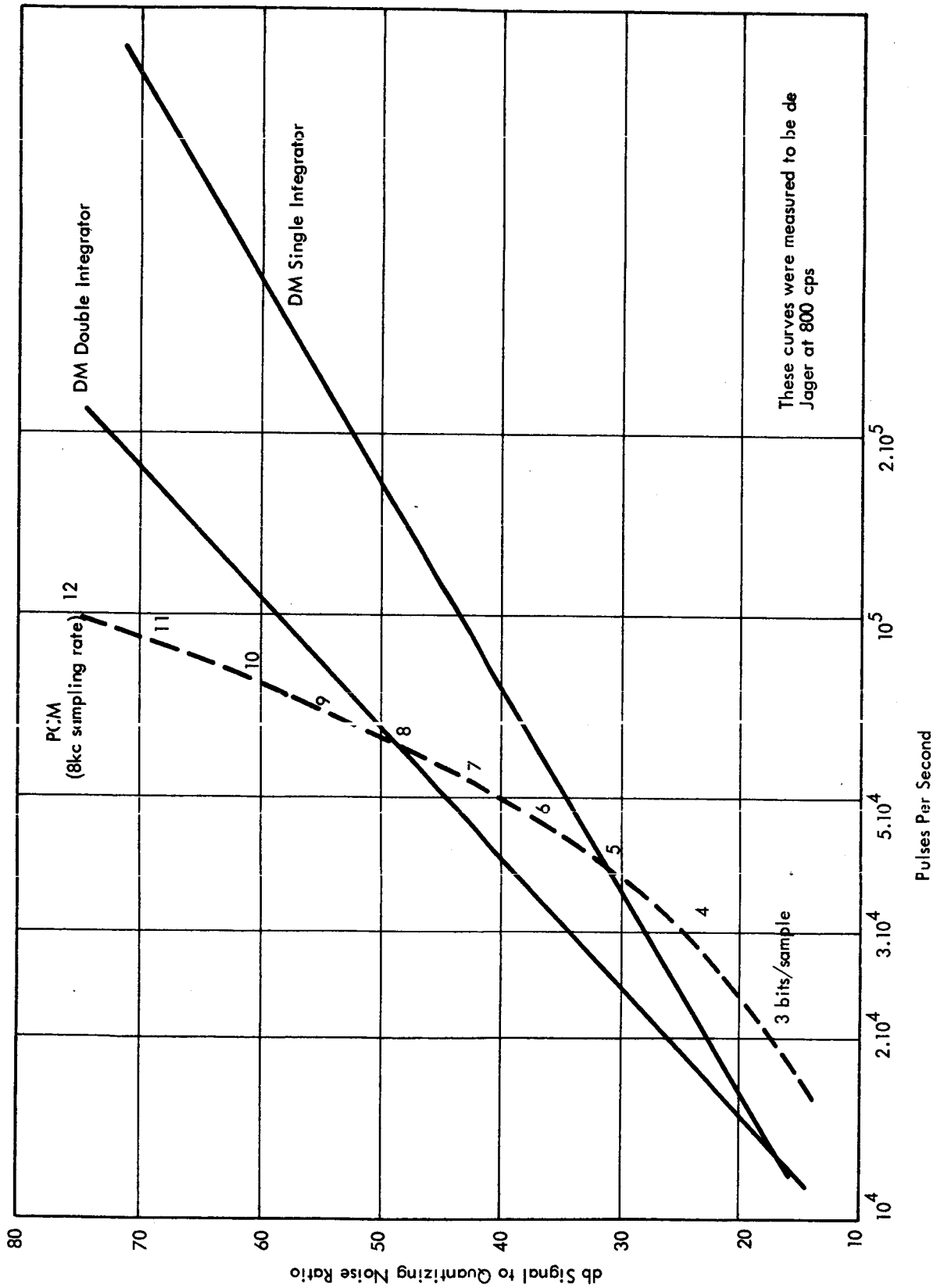


Figure F.2 Quantizing Signal to Noise Ratio vs Sampling Rate for DM and PCM

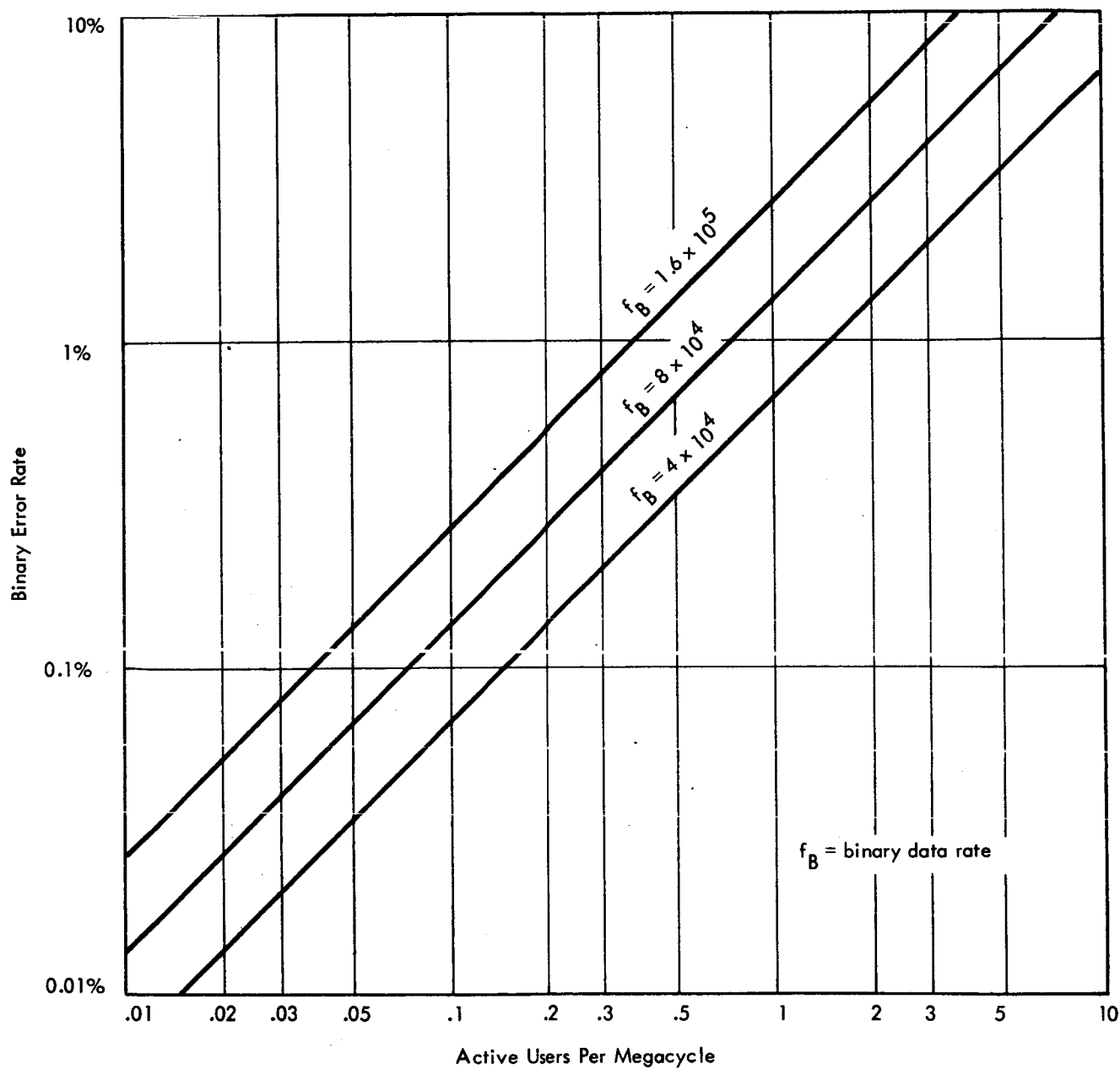


Figure F.3 Error Rate vs K/mc for an FT Matrix System

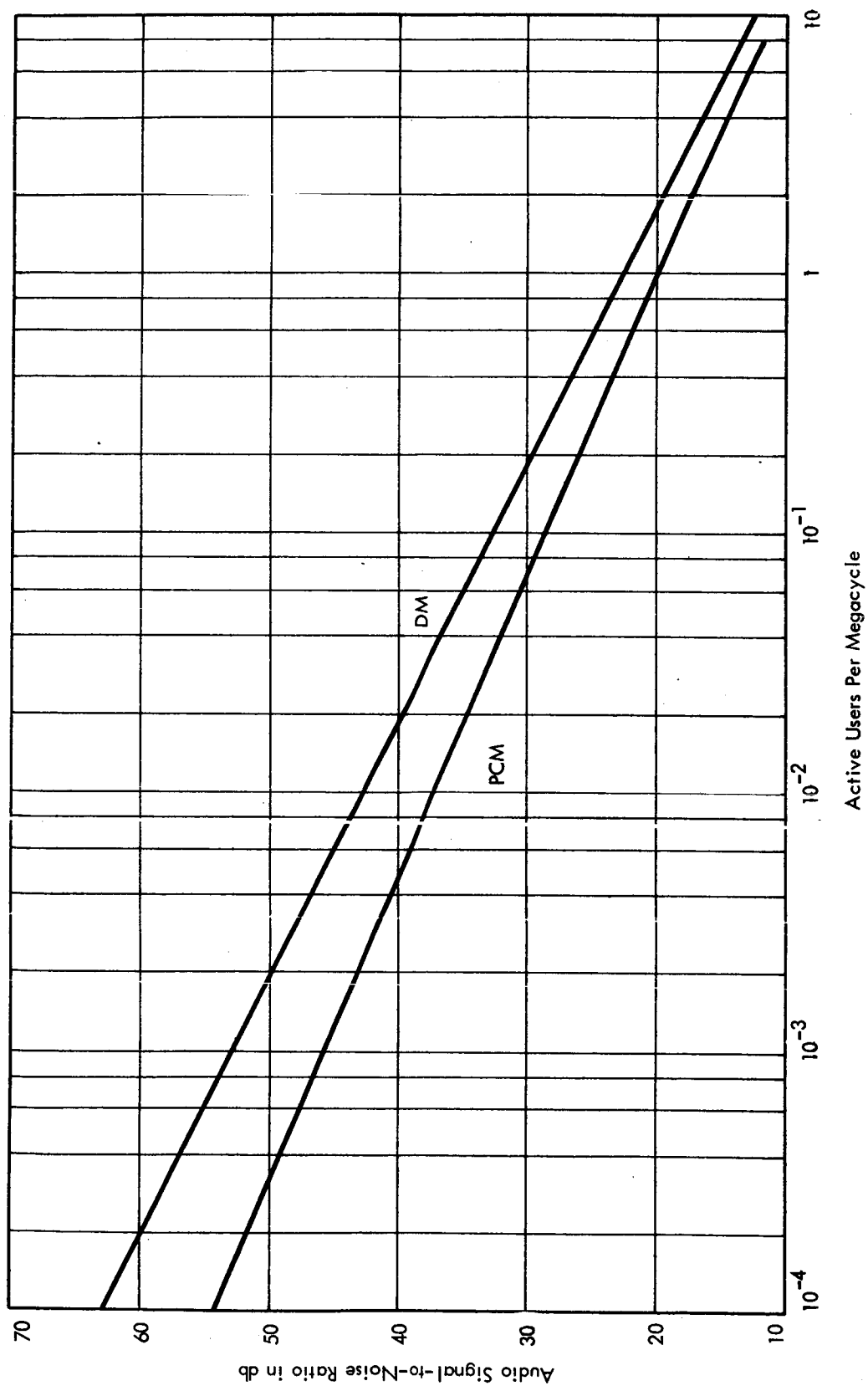
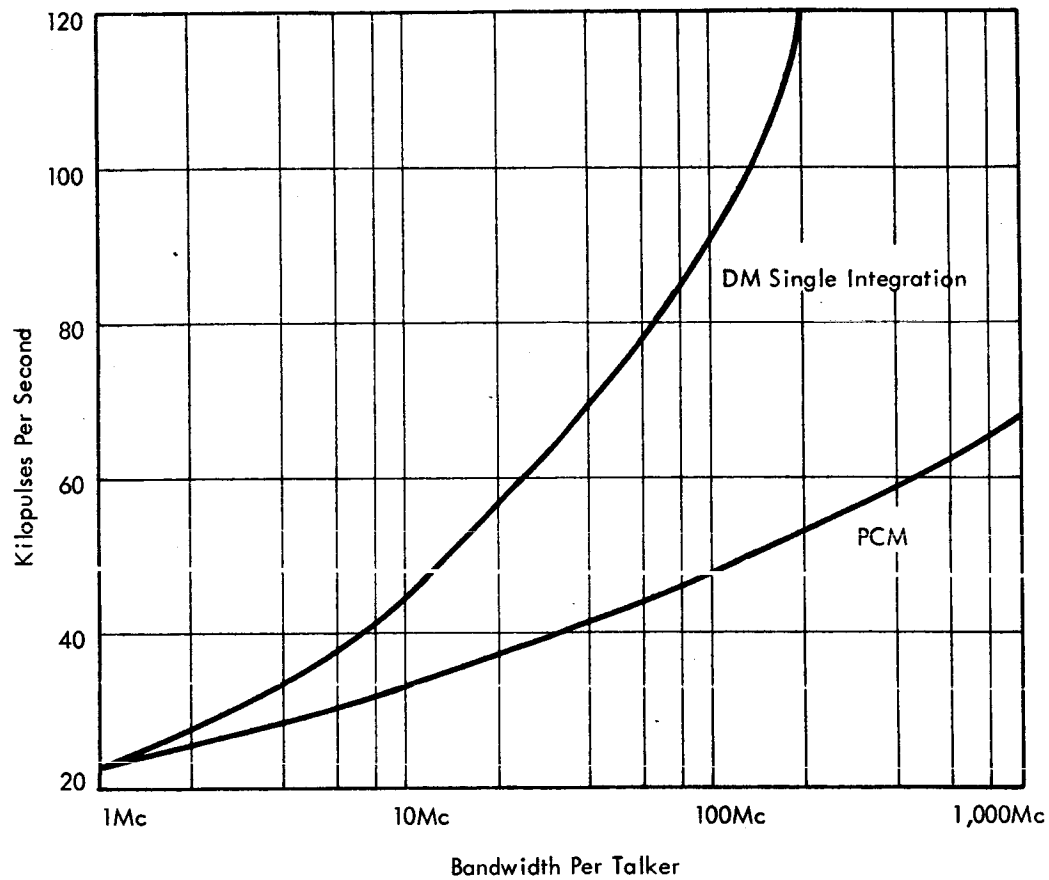


Figure F.4 Audio Signal to Noise Ratio vs K/mc for an FT Matrix System



*Figure F.5 Bandwidth Requirement as a Function of Error Rate fro PCM and Single Integration Delta Modulation*

## ADDENDUM

### CLUTTER CALCULATION

The peak signal power to noise power ratio at the output of a matched filter (or correlation receiver) is derived in this addendum. Analysis is general in that it is applicable to general classes of pseudo-noise signals.

When this signal-to-noise ratio is combined with the assumption that the clutter is a white Gaussian process, mathematical expressions for the error probability can be obtained. If the clutter is a random process consisting of an ensemble of Bernoulli random variable then exact calculations of the error probability show that the Gaussian error probability is a good approximation provided the signal-to-noise ratio is reduced by 2 db which represents a clipping loss. These results were obtained at IBM by using computer simulation.

The general mathematical expression for the  $n^{\text{th}}$  signal is,

$$z_n(t) = A_n \exp \{j[\omega_o t + n\Delta\omega t + \phi_n(t)]\}; \quad 0 \leq t \leq T \quad (1)$$

$$n = 1, 2, \dots, M$$

where

$$T = \text{time duration of signal}$$

$$\Delta\omega = \frac{2\pi}{T} = \text{frequency shift}$$

$$\omega_o = \text{carrier (or IF) frequency}$$

$$\phi_n(t) = \text{pseudo random phase modulation.}$$

This angle can contain a message component as well, which varies at a much slower rate. Of particular interest is,

$$\exp \{j\phi_n(t)\} = (1, -1, \dots, -1, -1, 1) = N \text{ bit pseudo random signal} \quad (2)$$

Then,

$$\Delta T = \frac{T}{N} = \text{time duration of pseudo-noise bit}$$

$$W = \frac{1}{2\Delta T} = \frac{N}{2T} = \text{ideal low-pass bandwidth.}$$

The received signal has the form,

$$z_R(t) = z_n(t) + z_c(t) + n(t) \quad (3)$$

$$z_c(t) = \text{clutter signal}$$

$$n(t) = \text{complex white gaussian noise process of spectral density } N_o \text{ watts per cps.}$$

The clutter is given by,

$$z_c(t) = \sum_{\substack{p \\ p \neq n}} A_p \exp \{j[(\omega_o + p\Delta\omega)(t - \tau_p) + \phi_p(t - \tau_p)]\} \quad (4)$$

The output of the matched filter is,

$$Z_n(\tau) = \int_0^T z_n(t) z_n^*(t+\tau) dt + \int_0^T z_c(t) z_n^*(t+\tau) dt + \int_0^T n(t) z_n^*(t+\tau) dt \quad (5)$$

where \* means the complex conjugate of the function.

At the instant of match the output of a matched filter (or active correlator) is given by,

$$Z_n(0) = \int_0^T |z_n(t)|^2 dt + \int_0^T z_c(t) z_n^*(t) dt + \int_0^T n(t) z_n^*(t) dt \quad (6)$$

The predetection signal-to-noise (plus clutter) power ratio at the instant of match is,

$$\eta^2 = \frac{\left| \int_0^T |z_n(t)|^2 dt \right|^2}{\frac{1}{2} \left| \int_0^T [z_c(t) + n(t)] z_n^*(t) dt \right|^2} \quad (7)$$

The peak energy obtained at the output of the matched filter is given by,

$$E_n(\text{peak}) = \int_0^T |z_n(t)|^2 dt = \int_0^T A_n^2 dt = A_n^2 T \quad (8)$$

Thus,

$$\eta^2 = \frac{2(A_n^2 T)^2}{\left| \int_0^T [z_c(t) + n(t)] z_n^*(t) dt \right|^2} \quad (9)$$

It is now necessary to calculate the denominator of Equation (9), for example, the clutter plus thermal noise power at the output of the matched filter. (From here on the term matched filter will mean any correlation device.)

From Equations (1), (4), and (5) we have,

$$Z_n = \sum_{\substack{p \\ p \neq n}} A_n A_p \exp\{j\Theta_p\} \int_0^T \exp\{j[(p-n)\Delta\omega t + \phi_p(t - \tau_p) - \phi_n(t)]\} dt \\ + \int_0^T n(t) A_n \exp\{-j[\omega_0 t + n\Delta\omega t + \phi_n(t)]\} dt \quad (10)$$

where  $\Theta_p = (\omega_0 + p\Delta\omega) \tau_p$  is an arbitrary (constant) phase angle. Let,

$$\Phi_{np} = \frac{1}{T} \int_0^T \exp\{j[(p-n)\Delta\omega t + \phi_p(t - \tau_p) - \phi_n(t)]\} dt \quad (11)$$

and

$$\Phi_N = \int_0^T n(t) A_n \exp\{-j[\omega_0 t + n\Delta\omega t + \phi_n(t)]\} dt \quad (12)$$

Substituting Equations (11) and (12) into Equation (10) yields,

$$Z_n = T \sum_{\substack{p \\ p \neq n}} A_n A_p \Phi_{np} \exp j\Theta_p + \Phi_N \quad (13)$$

It is now necessary to compute  $|Z_n|^2$ , the mean square noise plus clutter;

$$\overline{|Z_n|^2} = (A_n T)^2 \sum_{\substack{p \\ p \neq n}} \sum_{\substack{r \\ r \neq n}} A_p A_r \overline{\Phi_{np} \Phi_{nr}^*} \overline{\exp j(\Theta_p - \Theta_r)} \\ + \overline{|\Phi_N|^2} + T A_n \sum_{\substack{p \\ p \neq n}} A_p \overline{\Phi_N^* \Phi_{np}} \overline{\exp j\Theta_p} \\ + T A_n \sum_{\substack{r \\ r \neq n}} A_r \overline{\Phi_{nr}^* \Phi_N} \overline{\exp (-j\Theta_n)} \quad (14)$$

Now,

$$\overline{\exp j (\Theta_p - \Theta_r)} = 1 \quad p = r \quad (15)$$

$$= 0 \quad p \neq r$$

$$\overline{\Phi_N} = 0 \quad (16)$$

Thus, the total noise power is

$$|Z_n|^2 = (A_n T)^2 \sum_{\substack{p \\ p \neq n}} A_p^2 \overline{|\Phi_{np}|^2} + |\overline{\Phi_N}|^2 \quad (17)$$

where

$$|\Phi_N|^2 = \left| \int_0^T A_n \exp \{ -j [\omega_0 + n \Delta \omega] t + \phi_n(t) \} \cdot n(t) dt \right|^2 \quad (18)$$

The complex noise has the form

$$n(t) = A_N(t) \exp \{ j [\omega_0 t + \Theta_N(t)] \} \quad (19)$$

$$= [X_N(t) + j Y_N(t)] \exp j \omega_0 t \quad (20)$$

where

$$\overline{X_N^2} = \overline{Y_N^2} = \frac{1}{2} \overline{n^2} = \sigma_N^2 = \text{thermal noise power} \quad (21)$$

The functions  $\{X_N(t), Y_N(t)\}$  are Hilbert transforms of each other.

Thus,

$$|\overline{\Phi_N}|^2 = 2 \left| \int_0^T X_N(t) A_n \exp \{ -j [n \Delta \omega t + \phi_n(t)] \} dt \right|^2 \quad (22)$$

If we now represent Equation (22) by samples at points separated

by  $t = 1/W$ , (independent sample points of the Hilbert component) and

replace the integral by a sum we have,

$$\begin{aligned} |\overline{\Phi_N}|^2 &= 2 \left( \frac{A_n}{W} \right)^2 \sum_{i=1}^{WT} \sum_{q=1}^{WT} \exp \{ -j[n\Delta\omega(t_i - t_q)] \} \\ &\quad \cdot \exp \{ -j[\phi_n(t_i) - \phi_n(t_q)] \} \overline{X_{N_i}(t_i)X_{N_q}(t_q)} \end{aligned} \quad (23)$$

where,

$$\begin{aligned} \overline{X_{N_i}(t_i)X_{N_q}(t_q)} &= \sigma_N^2 = W N_o ; i = q \\ &= 0 ; i \neq q \end{aligned} \quad (24)$$

Hence,

$$|\overline{\Phi_N}|^2 = 2(A_n^2 T)N_o \quad (25)$$

From Equations (17) and (25) we have for the noise power,

$$|\overline{Z_n}|^2 = (A_n T)^2 \sum_{\substack{p \\ p \neq n}} A_p^2 |\overline{\Phi_{np}}|^2 + 2(A_n^2 T)N_o \quad (26)$$

where,

$$|\overline{\Phi_{np}}|^2 = \frac{1}{T^2} \left| \int_0^T \exp \{ j[(p-n)\Delta\omega t + \phi_p(t - \tau_p) - \phi_n(t)] \} dt \right|^2 \quad (27)$$

Equation (27) is the Ambiguity Function. The signal-to-noise ratio at the output of the matched filter is,

$$\eta_n^2 = \frac{1}{\frac{1}{2} \sum_{\substack{p \\ p \neq n}} \left( \frac{A_p}{A_n} \right)^2 |\overline{\Phi_{np}}|^2 + \frac{N_o}{A_n^2 T}} \quad (28)$$

If the clutter is zero we obtain the standard thermal noise, signal-to-noise ratio  $\eta^2 = \frac{A_n^2 T}{N_o} = \frac{E_n(\text{peak})}{N_o} = \frac{2E_n}{N_o}$ , where  $E_n$  is

the average signal energy. The term to the left is the clutter contributed at the output of the  $n^{\text{th}}$  matched filter (or after active correlation and filtering) by the common channel signals.

Let us now assume that the signal addresses are binary signals which take on positive and negative values and that the frequency shift is zero (i. e.,  $\Delta\omega = 0$ ). Then from Equation (27)

$$\overline{|\Phi_{np}|^2} = \frac{1}{T^2} \left| \int_0^T \exp\{j\phi_n(t - \tau_p)\} \cdot \exp\{-j\phi_n(t)\} dt \right|^2 \quad (29)$$

At the sample points the integrand is itself a binary signal. If each signal is a Bernoulli sequence the product is also a Bernoulli sequence. The integrand contains  $(N - \tau_p)$  bits where  $N$  is the total number of bits and  $\tau_p$  can be assumed to be an integer random variable which has a flat distribution. Changing Equation (29) from an integral to a sum we find that for a fixed value of  $\tau_p$ ,

Equation (29) for a fixed value of  $\tau_p$  is simply

$$\overline{|\Phi_{np}(\tau_p)|^2} = \left(\frac{\Delta T}{T}\right)^2 [N - \tau_p] \quad (30)$$

where  $T$  is the duration of the signal and  $\Delta T = T/N$

If Equation (30) is now averaged over variations in  $\tau_p$  we have

$$\begin{aligned} \overline{|\Phi_{np}|^2} &= \left(\frac{\Delta T}{T}\right)^2 \sum_{\tau_p=1}^N \frac{2}{N} (N - \tau_p) \\ &= \left(\frac{\Delta T}{T}\right)^2 (N-1) \approx N \left(\frac{\Delta T}{T}\right)^2 ; N \gg 1. \end{aligned} \quad (31)$$

Since,  $\frac{T}{\Delta T} = N$ , we have

$$|\Phi_{np}|^2 = \frac{1}{N} \quad (32)$$

Thus, in this special case the peak signal-to-noise power ratio is,

$$\begin{aligned} \eta^2 &= \frac{1}{\frac{1}{2N} \sum_{\substack{p \\ p \neq n}} \left( \frac{A_p}{A_n} \right)^2 + \left( \frac{N_o}{A_n^2 T} \right)} \\ &= \frac{1}{\frac{1}{2W} \sum_{\substack{p \\ p \neq n}} \frac{A_p^2}{2} + N_o} \end{aligned} \quad (33)$$

The left term in the denominator shows the average clutter energy spread over the RF bandwidth  $2W$ . It is evident that as the bandwidth is increased the mutual clutter is spread over a broadband and its effect is decreased until thermal noise becomes the limiting factor. Thus, when  $W \rightarrow \infty$

$$\eta_{\infty}^2 = \frac{A_n^2 T}{N_o} = \frac{2E}{N_o} \quad (34)$$

as required.

Let  $P$  be the total satellite power output. Then

$$P = \frac{A_n^2}{2} + \sum_{\substack{p=1 \\ p \neq n}}^K \frac{A_p^2}{2} \quad (35)$$

Substituting Equation (35) for the clutter gives

$$\begin{aligned}
 \eta^2 &= \frac{A_n^2 T}{\frac{1}{2W} \left( P - \frac{A_n^2}{2} \right) + N_o} \\
 &= \frac{2 \cdot \left( \frac{A_n^2}{2P} \right) T}{\frac{1}{2W} \left( 1 - \frac{A_n^2}{2P} \right) + \frac{N_o}{P}}
 \end{aligned} \tag{36}$$

The term  $\frac{P}{N_o}$  is a constant of the satellite system.

The channel capacity of the satellite system as the bandwidth becomes very large is,

$$\begin{aligned}
 C &= W \ln \left( 1 + \frac{P}{W N_o} \right) \text{ NITS} \\
 C &= C_\infty = \frac{P}{N_o} \text{ as } W \rightarrow \infty
 \end{aligned} \tag{37}$$

Hence,

$$\eta^2 = \frac{4W C_\infty T \left( \frac{A_n^2}{2P} \right)}{C_\infty \left( 1 - \frac{A_n^2}{2P} \right) + 2W} \tag{38}$$

Note that the effective bandwidth is,

$$2W_e = \frac{2W C_\infty}{C_\infty \left( 1 - \frac{A_n^2}{2P} \right) + 2W} \tag{39}$$

and

$$\eta^2 = 4W_e T \left( \frac{A_n^2}{2P} \right) \quad (40)$$

If all signals are multiplexed at equal power we have, from Equation (35),

$$\frac{A_n^2}{2P} = \frac{1}{K}$$

and Equation (40) becomes

$$\eta^2 = \frac{4W_e T}{K} \quad (41)$$

In particular, if  $2W \gg C_\infty$ ,

$$\eta_\infty = \frac{2C_\infty T}{K} \quad (42)$$

and communication is thermal noise limited. On the other hand if  $C_\infty \gg 2W$  and  $K \gg 1$ ,

$$\eta_W = \frac{4WT}{K} \quad (43)$$

and communications is clutter limited.

Another important class of signals is the PN frequency shifted alphabet. Here an address is a frequency shift; each signal has the same pseudo-noise binary phase modulation. In this case,

$$\overline{|\Phi_{np}|^2} = \frac{1}{T^2} \left| \int_0^T \exp \{ j[(p-n)\Delta\omega t + \phi(t-\tau_p) - \phi(t)] \} dt \right|^2 \quad (44)$$

The clutter power is given by the ambiguity function.

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